Double Integrals in Polar Coordinates

For problems 1 & 2 evaluate the integral over the given region. 1. $\iint_{D} 4xy^2 dA$, *D* is the region between $x^2 + y^2 = 4$ and $x^2 + y^2 = 1$ and in the 3rd quadrant.

2.
$$\iint_{D} \frac{1}{\sqrt{2x^2 + 2y^2 + 1}} dA$$
, *D* is the disk given by $x^2 + y^2 = 2$.

3. Find the volume of the solid that is bounded by $y = 7 - 4x^2 - 4z^2$ and $y = 2x^2 + 2^2 - 5$. Note that you will have to use a *modified* version of polar coordinates to do this problem.

4. Use a double integral to derive the formula for the area of a circle of radius *a*.

5. Evaluate $\int_{-2}^{0} \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \sqrt{x^2 + y^2} \, dx \, dy$ by converting the integral into polar coordinates.

Triple Integrals

For problems 6 – 9 evaluate the given integral.

6.
$$\int_{1}^{-2} \int_{0}^{x^{2}} \int_{-y}^{y} y \mathbf{e}^{x^{7}} dz dy dx$$

7. $\iint_{E} 36z \, dV$ where *E* is the solid bounded by the planes 5x + 2y + z = 10, x = 0, y = 0, and z = 0. In other words *E* is the solid that lies beneath 5x + 2y + z = 10 and in the first octant.

8. $\iiint_E 4z \, dV$ where *E* is the solid that lies between 2x + 6y + 5z = 30 and 4x + y + z = 4 and is in front of the region in the *yz*-plane bounded by $z = y^2$ and $z = \sqrt{y}$.

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9.
$$\iiint_E yz \, dV$$
 where *E* is the solid that is behind $x = 5 - 2y^2 - 2z^2$ and in front of $x = y^2 + z^2 - 7$.

10. Use a triple integral to find the volume of the solid *E* used in problem 8.

Triple Integrals with Cylindrical Coordinates

For problems 11 – 13 you must use cylindrical coordinates to do the problem.

11.
$$\iiint_E x \, dV$$
 where *E* is the solid that lies inside $y^2 + z^2 = 16$, in front of $x = 4 - y^2 - z^2$ and behind $x = 5$.

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12. Find the volume of the solid *E* that is inside $x^2 + y^2 = 1$ below $z = \sqrt{x^2 + y^2}$ and above $z = -\sqrt{3x^2 + 3y^2}$.

13. Use a triple integral to find a formula for the volume of a cylinder of radius *a* and height *h*.