1. (2 pts) The limits are : $\pi \le \theta \le \frac{3\pi}{2},\ \ 1 \le r \le 2$

$$\iint\limits_{D}\iint\limits_{D} 4xy^2\, dA = \int_{\pi}^{\frac{3}{2}\pi}\int_{1}^{2} 4(r\cos\theta)(r\sin\theta)^2\, r\,dr\,d\theta = \int_{\pi}^{\frac{3}{2}\pi}\int_{1}^{2} 4r^4\cos\theta\sin^2\theta\, dr\,d\theta$$

$$= \int_{\pi}^{\frac{3}{2}\pi}\left(\tfrac{4}{5}r^5\cos\theta\sin^2\theta\right)\Big|_{1}^{2}\, d\theta = \int_{\pi}^{\frac{3}{2}\pi}\tfrac{124}{5}\cos\theta\sin^2\theta\, d\theta$$

$$= \left(\tfrac{124}{15}\sin^3\theta\right)\Big|_{\pi}^{\frac{3}{2}\pi} = \boxed{-\tfrac{124}{15}}$$

7. (2 pts) Here's a sketch of the plane and the region *D* in the *xy*-plane.
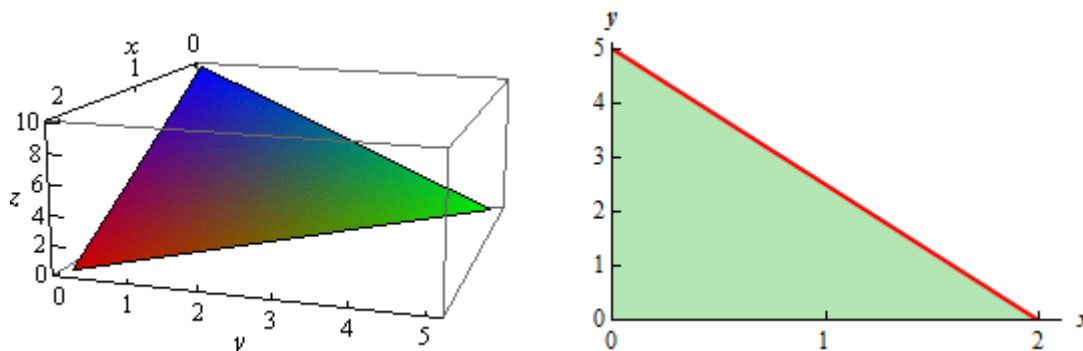
The equation for the line in the *xy*-plane is $5x + 2y = 10$ which we get by setting $z = 0$ in the equation of the plane. The limits for *E* are then : $0 \le x \le 2,\ 0 \le y \le 5 - \tfrac{5}{2}x,\ 0 \le z \le 10 - 5x - 2y$. The integral is then,

$$\iiint\limits_{E} 36z\, dV = \int_{0}^{2}\int_{0}^{5-\frac{5}{2}x}\int_{0}^{10-5x-2y} 36z\, dz\, dy\, dx = \int_{0}^{2}\int_{0}^{5-\frac{5}{2}x} 18(10-5x-2y)^2\, dy\, dx$$

$$= \int_{0}^{2} -3(10-5x-2y)^3\Big|_{0}^{5-\frac{5}{2}x}\, dx = \int_{0}^{2} 3(10-5x)^3\, dx = -\tfrac{3}{20}(10-5x)^4\Big|_{0}^{2} = \boxed{1500}$$

10. (3 pts)
A sketch for this problem is shown in #8 and from #8 the limits are,

$$0 \le y \le 1,\quad y^2 \le z \le \sqrt{y},\quad 1 - \tfrac{1}{4}y - \tfrac{1}{4}z \le x \le 15 - 3y - \tfrac{5}{2}z$$

The volume is then,

$$V = \iiint\limits_{E} dV = \int_{0}^{1}\int_{y^2}^{\sqrt{y}}\int_{1-\frac{1}{4}y-\frac{1}{4}z}^{15-3y-\frac{5}{2}z} dx\, dz\, dy = \int_{0}^{1}\int_{y^2}^{\sqrt{y}} 14 - \tfrac{11}{4}y - \tfrac{9}{4}z\, dz\, dy$$

$$= \int_{0}^{1} 14\sqrt{y} - \tfrac{9}{8}y - \tfrac{11}{4}y^{\frac{3}{2}} - 14y^2 + \tfrac{11}{4}y^3 + \tfrac{9}{8}y^4\, dz = \boxed{\tfrac{47}{12}}$$

11. (3 pts) Here are a couple of figures to help us see the region. Note that I have the *x* coordinates in the opposite direction (negative to front and positive to back) in order to better view the region. The front surface is a plane and the back surface is a paraboloid as we can see in the left figure. The walls of the region are the cylinder as shown in the figure to the right. So the region is the portion that is inside the cylinder, behind the plane and in front of the paraboloid.
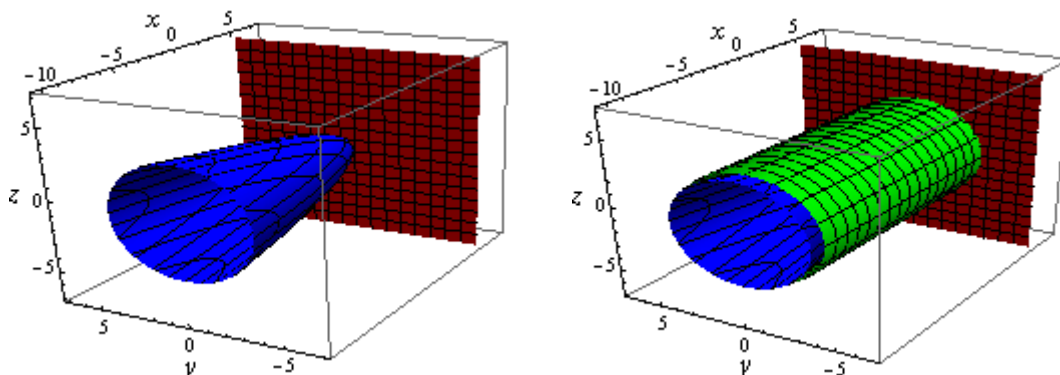
The limits for the integral are : $4 - r^2 \le x \le 5$, $0 \le r \le 4$, $0 \le \theta \le 2\pi$. We'll also be using the polar coordinates : $y = r\sin\theta$, $z = r\cos\theta$, $y^2 + z^2 = r^2$. The integral is,

$$\iiint_E x\, dV = \int_0^{2\pi}\int_0^4\int_{4-r^2}^5 xr\, dx\, dr\, d\theta = \tfrac{1}{2}\int_0^{2\pi}\int_0^4 9r + 8r^3 - r^5\, dr\, d\theta$$

$$= \tfrac{1}{2}\int_0^{2\pi} -\tfrac{296}{3}\, d\theta = \boxed{-\tfrac{296}{3}\pi}$$

Not Graded

2. The limits are : $0 \le \theta \le 2\pi$, $0 \le r \le \sqrt{2}$

$$\iint_D \frac{1}{\sqrt{2x^2 + 2y^2 + 1}}\, dA = \int_0^{2\pi}\int_0^{\sqrt{2}} \frac{r}{\sqrt{2r^2 + 1}}\, dr\, d\theta = \int_0^{2\pi} \tfrac{1}{2}\sqrt{2r^2 + 1}\Big|_0^{\sqrt{2}}\, d\theta$$

$$= \int_0^{2\pi} \tfrac{1}{2}\left(\sqrt{5} - 1\right) d\theta = \boxed{\pi\left(\sqrt{5} - 1\right)}$$

3. We have two paraboloids centered on the *x*-axis. The first one starts at 7 and opens backwards and the second starts at -5 and opens towards the front. They will intersect at,

$$7 - 4x^2 - 4z^2 = 2x^2 + 2^2 - 5 \quad \Rightarrow \quad 6x^2 + 6z^2 = 12 \quad \Rightarrow \quad x^2 + z^2 = 2$$

So, *D* is a disk of radius $\sqrt{2}$ centered at the origin. Now, we'll need to use the following "version" of polar coordinates : $x = r\sin\theta$, $y = r\cos\theta$. Using this means that $x^2 + z^2 = r^2$ and so the integral for the volume will then be,

$$V = \iint_D 7 - 4x^2 - 4z^2 - \left(2x^2 + 2z^2 - 5\right) dA = \iint_D 12 - 6x^2 - 6z^2 \, dA$$

$$= \int_0^{2\pi} \int_0^{\sqrt{2}} \left(12 - 6r^2\right) r \, dr \, d\theta = \int_0^{2\pi} \left(6r^2 - \tfrac{3}{2}r^4\right)\Big|_0^{\sqrt{2}} \, d\theta = \int_0^{2\pi} 6 \, d\theta = \boxed{12\pi}$$

4. Not much to this problem.

$$A = \iint_D dA = \int_0^{2\pi} \int_0^a r \, dr \, d\theta = \tfrac{1}{2}\int_0^{2\pi} r^2 \Big|_0^a \, d\theta = \tfrac{1}{2}\int_0^{2\pi} a^2 \, d\theta = \pi a^2$$

5. Let's first get the limits for x and y : $-2 \le y \le 0,\ -\sqrt{4-y^2} \le x \le \sqrt{4-y^2}$. The limits on x tell us that we have some portion of a circle of radius 2 centered at the origin and the limits on y tell us that it will in fact be the lower portion of the circle and so the limits for D in terms of polar coordinates will be : $0 \le r \le 2,\ \pi \le \theta \le 2\pi$. The integral is then (don't forget that for the differential we have $dxdy = dA = rdrd\theta$),

$$\int_{-2}^0 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \sqrt{x^2 + y^2} \, dx \, dy = \int_\pi^{2\pi} \int_0^2 r^2 \, dr \, d\theta = \int_\pi^{2\pi} \tfrac{1}{3} r^3 \Big|_0^2 \, d\theta = \int_\pi^{2\pi} \tfrac{8}{3} \, d\theta = \boxed{\tfrac{8}{3}\pi}$$

6.

$$\int_1^{-2} \int_0^{x^2} \int_{-y}^{y} y e^{x^7} \, dz \, dy \, dx = \int_1^{-2} \int_0^{x^2} zy e^{x^7} \Big|_{-y}^{y} \, dy \, dx = \int_1^{-2} \int_0^{x^2} 2y^2 e^{x^7} \, dy \, dx = \int_1^{-2} \tfrac{2}{3} y^3 e^{x^7} \Big|_0^{x^2} \, dx$$

$$= \int_1^{-2} \tfrac{2}{3} x^6 e^{x^7} \, dx = \tfrac{2}{21} e^{x^7} \Big|_1^{-2} = \boxed{\tfrac{2}{21}\left(e^{-128} - e\right) = -0.258884}$$

8. Here's a sketch of the two planes as well as the region in *xz*-plane
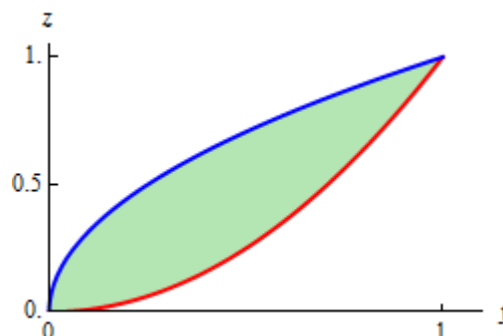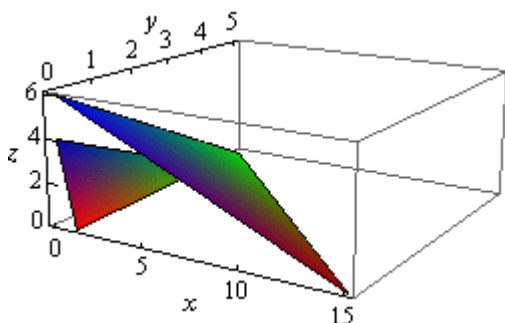
Because the region D is in the *yz*-plane we'll need the equations of the planes in terms of $x = f(y,z)$. Also we can integrate y first and then z or the other way around. Because the equations for D are in terms of z we'll integrate that first. So, the limits for the integral are,

$$0 \le y \le 1, \quad y^2 \le z \le \sqrt{y}, \quad 1 - \tfrac{1}{4}y - \tfrac{1}{4}z \le x \le 15 - 3y - \tfrac{5}{2}z$$

The integral is then,

$$\iiint\limits_{E} 4z\,dV = \int_0^1 \int_{y^2}^{\sqrt{y}} \int_{1-\frac{1}{4}y-\frac{1}{4}z}^{15-3y-\frac{5}{2}z} 4z\,dx\,dz\,dy = \int_0^1 \int_{y^2}^{\sqrt{y}} 56z - 11yz - 9z^2\,dz\,dy$$

$$= \int_0^1 28y - 3y^{\frac{3}{2}} - \tfrac{11}{2}y^2 - 28y^4 + \tfrac{11}{2}y^5 + 3y^6\,dz = \boxed{\tfrac{2819}{420}}$$

9. We'll need the intersection of these two surfaces to determine D.

$$5 - 2y^2 - 2z^2 = y^2 + z^2 - 7 \quad \Rightarrow \quad y^2 + z^2 = 4$$

So, D is the unit circle in the yz-plane. The limits on x are $y^2 + z^2 - 7 \le x \le 5 - 2y^2 - 2z^2$ and here is the first integration.

$$\iiint\limits_{E} yz\,dV = \iint\limits_{D} \int_{y^2+z^2-7}^{5-2y^2-2z^2} yz\,dx\,dA = \iint\limits_{D} yz\left(12 - 3y^2 - 3z^2\right)dA$$

Now, the double integral looks like it would be best done in polar coordinates so we'll use them in the form : $y = r\sin\theta,\ z = r\cos\theta,\ y^2 + z^2 = r^2$. The limits are : $0 \le r \le 3,\ 0 \le \theta \le 2\pi$.

$$\iiint\limits_{E} yz\,dV = \iint\limits_{D} r^2\cos\theta\sin\theta\left(12 - 3r^2\right)dA = \int_0^{2\pi}\int_0^2 \tfrac{1}{2}\sin(2\theta)\left(12 - 3r^2\right)r^2\left(r\right)dr\,d\theta$$

$$= \int_0^{2\pi} \tfrac{1}{2}\sin(2\theta)(16)\,d\theta = \boxed{0}$$

12. Here are a couple of figures to help us see the region. The upper and lower surfaces are cones as shown to the left. The walls of the region are the cylinder as shown in the figure to the right. So the region is the portion that is inside the cylinder and between the two cones.
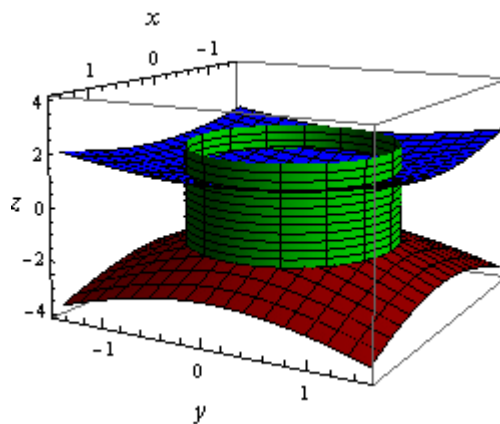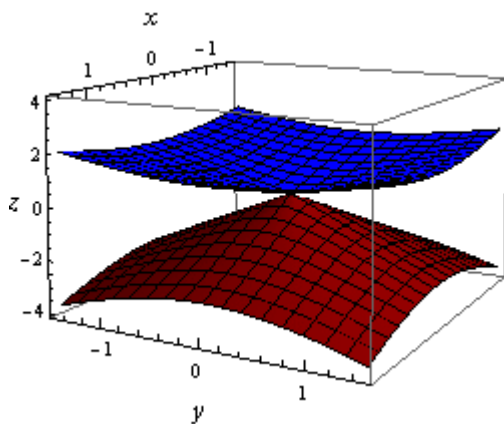
The limits for the integral are : $-\sqrt{3}\,r \le z \le r,\ \ 0 \le r \le 1,\ \ 0 \le \theta \le 2\pi$. The integral is,

$$V = \iiint\limits_{E} dV = \int_0^{2\pi}\int_0^1\int_{-\sqrt{3}\,r}^{r} r\,dz\,dr\,d\theta = \int_0^{2\pi}\int_0^1 \left(1 + \sqrt{3}\right)r^2\,dr\,d\theta$$

$$= \int_0^{2\pi} \tfrac{1}{3}\left(1 + \sqrt{3}\right)d\theta = \boxed{\tfrac{2}{3}\left(1 + \sqrt{3}\right)\pi}$$

13. This one isn't too bad once you think about it. Since we are inside the cylinder of radius *a* we know that the limits on *r* and θ will be,

$$0 \le \theta \le 2\pi \qquad\qquad 0 \le r \le a$$

Now, the height of *h* just tells us that we want the portion of the cylinder between the planes $z = 0$ and $z = h$. Therefore, the limits on *z* will be,

$$0 \le z \le h$$

The volume of the cylinder, using a triple integral is then,

$$V = \iiint\limits_{E} dV = \int_0^{2\pi} \int_0^a \int_0^h r \, dz \, dr \, d\theta = \int_0^{2\pi} \int_1^{\sqrt{5}} r \, h \, dr \, d\theta = \int_0^{2\pi} \tfrac{1}{2} a^2 \, h \, d\theta = \pi a^2 \, h$$