Math 2415 **Homework Set 7** **10 Points**

Triple Integrals with Spherical Coordinates

For problems 1 and 2 you must use spherical coordinates to do the problems.

1. $\iiint\limits_E 8x + 4y\,dV$ where *E* is the region below the sphere of radius 3 and inside the cone $\varphi = \frac{\pi}{3}$.

2. $\iiint\limits_E \mathbf{e}^{\left(x^2+y^2+z^2\right)^{\frac{3}{2}}}\,dV$ where *E* is the region that lies in the first octant and between the spheres of radius 1 and 4.

3. Evaluate the following integral by first converting to spherical coordinates.

$$\int_{-\sqrt{3}}^{0}\int_{0}^{\sqrt{3-x^2}}\int_{-\sqrt{9-x^2-y^2}}^{-\sqrt{2x^2+2y^2}} zx\,dz\,dy\,dx$$

Change of Variables

For problems 4 and 5 find the Jacobian of the transformation.

4. $x = uv - 5u^3 + 7v, \quad y = 3v - 8u$

5. $x = \mu\cos\alpha, \ y = \mu\sin\alpha$

For problems 6 – 8 find and graph the image of the set *R* under the given transformation. Note that the point of these examples is not necessarily to transform *R* into a "nice" region. Instead all we're trying to do is apply some transformations to regions.

6. *R* is the triangle with vertices (0, 0), (-2, 2), (6, 2) and the transformation is $x = 3v, \ y = \sqrt{2-u}$.

Note that for each side you'll want the range of possible *x* or *y* values to in order to get a range of possible *u* or *v* values that are needed for the transformed sides. This will make the sketch a little easier maybe……

7. *R* is the region bounded by $xy = 2, \ xy = 6, \ y = x, \ y = 4$ and the transformation is $x = u, \ y = \frac{2v}{u}$.

8. *R* is the disk given by $x^2 + y^2 \le 1$ and the transformation is $u = ax, \ v = by$.

9. If *R* is the parallelogram with vertices (2, -1), (-2, 1), (4, 5) and (0, 7) use the transformation $x = 2\left(u - 2v\right)$, $y = -\left(u + 12v\right)$ to convert the region and evaluate the integral $\iint\limits_R 3x - y\,dA$.

Continued on Back $\Rightarrow$

10. If *R* is the ellipse $x^2 + \frac{y^2}{4} = 1$ determine a transformation that will convert this into a circle of radius 1 and use that transformation to evaluate $\iint\limits_R xy\, dA$.

Note that you've already seen how to turn a circle of radius 1 into an ellipse so use that as a guide to determine this transformation.

Surface Area

For problems 11 and 12 find the area of the given surface.

11. The portion of the plane $8x + 4y + 3z = 24$ that lies in the first octant.

12. The portion of $z = 4x^2 + 4y^2 - 1$ that lies below $z = 11$.

13. In class I gave you the formula for the surface area of $z = f(x, y)$ that lies above a region *D* in the *xy*-plane. Use a modification of this formula to find the surface area of the portion of the surface $y = 6 + 7x + 2z^2$ that lies in front of the triangle in the *xz*-plane with vertices (0,0), (6,3) and (0,3).