

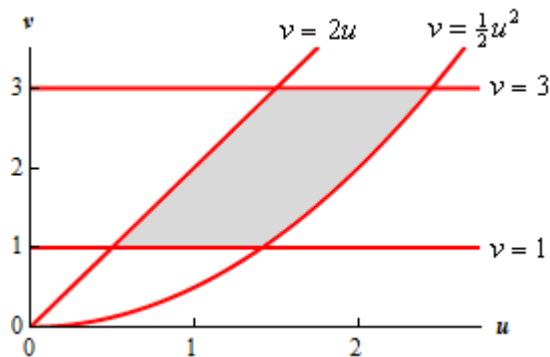
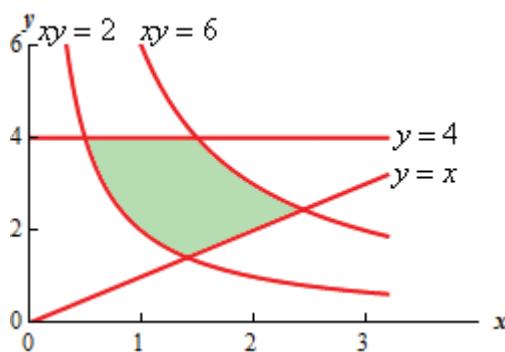
2. (2 pts) The limits are : $1 \leq \rho \leq 4$, $0 \leq \theta \leq \frac{\pi}{2}$, $0 \leq \varphi \leq \frac{\pi}{2}$. The integral is then,

$$\begin{aligned} \iiint_E e^{(x^2+y^2+z^2)^{\frac{3}{2}}} dV &= \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_1^4 \rho^2 e^{\rho^3} \sin \varphi d\rho d\theta d\varphi = \frac{1}{3}(e^{64} - e) \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \sin \varphi d\theta d\varphi \\ &= \frac{\pi}{6}(e^{64} - e) \int_0^{\frac{\pi}{2}} \sin \varphi d\varphi = \boxed{\frac{\pi}{6}(e^{64} - e)} \end{aligned}$$

4. (2 pts)

$$\begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} v-15u^2 & u+7 \\ -8 & 3 \end{vmatrix} = 3(v-15u^2) + 8(u+7) = \boxed{3v+8u+56-45u^2}$$

7. (3 pts) Here is a sketch of the original region and the transformed region. Transformation work is below.



Here is the work for transforming each of the sides.

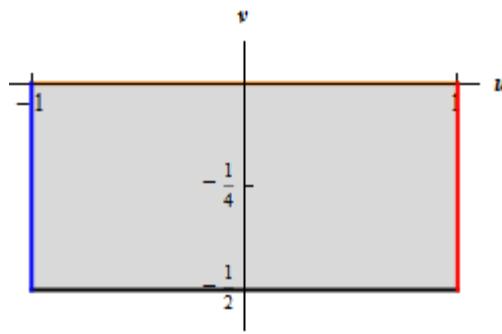
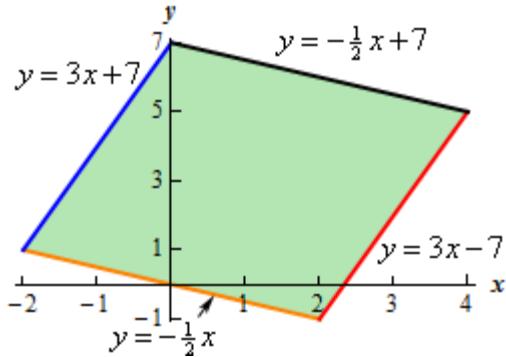
$$xy = 2 \quad : \quad (u)\left(\frac{2v}{u}\right) = 2 \quad \Rightarrow \quad v = 1$$

$$xy = 6 \quad : \quad (u)\left(\frac{2v}{u}\right) = 6 \quad \Rightarrow \quad v = 3$$

$$y = x \quad : \quad \frac{2v}{u} = u \quad \Rightarrow \quad v = \frac{1}{2}u^2$$

$$y = 4 \quad : \quad \frac{2v}{u} = 4 \quad \Rightarrow \quad v = 2u$$

9. (3 pts) Here are the sketches. Transformation work is below.



Here's the transformation work.

$$\begin{array}{lll}
 y = -\frac{1}{2}x & -(u+12v) = -\frac{1}{2}(2)(u-2v) & \Rightarrow \quad \underline{v=0} \\
 y = -\frac{1}{2}x + 7 & -(u+12v) = -\frac{1}{2}(2)(u-2v) + 7 & \Rightarrow \quad \underline{v=-\frac{1}{2}} \\
 y = 3x + 7 & -(u+12v) = 3(2)(u-2v) + 7 & \Rightarrow \quad \underline{u=-1} \\
 y = 3x - 7 & -(u+12v) = 3(2)(u-2v) - 7 & \Rightarrow \quad \underline{u=1}
 \end{array}$$

Next, we'll need the Jacobian.

$$\left| \begin{array}{cc} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{array} \right| = \left| \begin{array}{cc} 2 & -4 \\ -1 & -12 \end{array} \right| = -24 - (4) = -28$$

The integral is then,

$$\iint_R 3x - y \, dA = \iint_S 3[2(u-2v)] - [-(u+12v)] - 28 \, dA = 28 \int_{-\frac{1}{2}}^0 \int_{-1}^1 7u \, du \, dv = 28 \int_{-\frac{1}{2}}^0 0 \, dv = \boxed{0}$$

Not Graded

1. The limits are : $0 \leq \rho \leq 3$, $0 \leq \theta \leq 2\pi$, $0 \leq \varphi \leq \frac{\pi}{3}$. The integral is then,

$$\begin{aligned}
 \iiint_E 8x + 4y \, dV &= \int_0^{\frac{\pi}{3}} \int_0^{2\pi} \int_0^3 (8\rho \sin \varphi \cos \theta + 4\rho \sin \varphi \sin \theta) \rho^2 \sin \varphi \, d\rho \, d\theta \, d\varphi \\
 &= \int_0^{\frac{\pi}{3}} \int_0^{2\pi} \int_0^3 4(2 \cos \theta + \sin \theta) \rho^3 \sin^2 \varphi \, d\rho \, d\theta \, d\varphi \\
 &= \int_0^{\frac{\pi}{3}} \int_0^{2\pi} 81(2 \cos \theta + \sin \theta) \sin^2 \varphi \, d\theta \, d\varphi \\
 &= \int_0^{\frac{\pi}{3}} (81(2 \sin \theta - \cos \theta) \rho^3 \sin^2 \varphi) \Big|_0^{2\pi} \, d\varphi = \boxed{0}
 \end{aligned}$$

3. First let's get the limits on x , y , and z .

$$-\sqrt{3} \leq x \leq 0, 0 \leq y \leq \sqrt{3-x^2}, -\sqrt{9-x^2-y^2} \leq z \leq \sqrt{2x^2+2y^2}$$

The limits on x and y tell us that the region D in the original integral was the portion of the circle of radius $\sqrt{3}$ in the 2nd quadrant centered at the origin. The lower surface of the solid E is a sphere of radius 3 while the upper surface of E is a cone. From this information we can get the following two sets of limits.

$$0 \leq \rho \leq 3, \quad \frac{\pi}{2} \leq \theta \leq \pi$$

Now, we just need to find the limits on φ and we can get that from the cone.

$$\begin{aligned} z &= -\sqrt{2x^2+2y^2} = -\sqrt{2}\sqrt{x^2+y^2} \\ \rho \cos \varphi &= -\sqrt{2}\rho \sin \varphi \\ -\frac{1}{\sqrt{2}} &= \tan \varphi \quad \Rightarrow \quad \varphi = -0.61547 + \pi = 2.52611 \end{aligned}$$

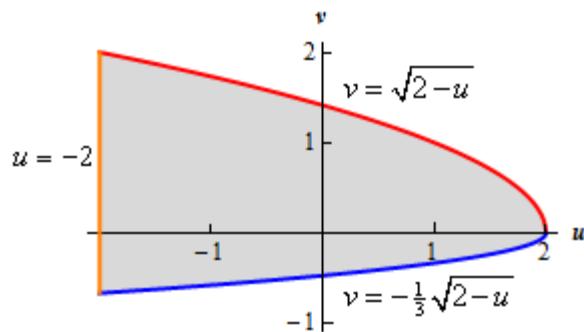
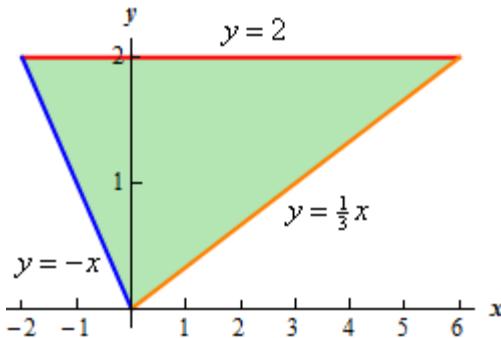
Recall that the φ must be in the range $0 \leq \varphi \leq \pi$ and so we had to add π to the answer from our calculator. Finally because this is a cone below the xy -plane the limits on φ must be $2.52611 \leq \varphi \leq \pi$. The integral is then,

$$\begin{aligned} \int_{-\sqrt{3}}^0 \int_0^{\sqrt{3-x^2}} \int_{-\sqrt{2x^2+2y^2}}^{-\sqrt{2x^2+2y^2}} zx \, dz \, dy \, dx &= \int_{2.52611}^{\pi} \int_{\frac{\pi}{2}}^{\pi} \int_0^3 (\rho \cos \varphi)(\rho \sin \varphi \cos \theta) \rho^2 \sin \varphi \, d\rho \, d\theta \, d\varphi \\ &= \int_{2.52611}^{\pi} \int_{\frac{\pi}{2}}^{\pi} \int_0^3 \rho^4 \sin^2 \varphi \cos \varphi \cos \theta \, d\rho \, d\theta \, d\varphi \\ &= \frac{243}{5} \int_{2.52611}^{\pi} \int_{\frac{\pi}{2}}^{\pi} \sin^2 \varphi \cos \varphi \cos \theta \, d\theta \, d\varphi \\ &= -\frac{243}{5} \int_{2.52611}^{\pi} \sin^2 \varphi \cos \varphi \, d\varphi = -\frac{81}{5} \sin^3 \varphi \Big|_{2.52611}^{\pi} = \boxed{3.11773} \end{aligned}$$

5.

$$\begin{vmatrix} \frac{\partial x}{\partial \mu} & \frac{\partial x}{\partial \alpha} \\ \frac{\partial y}{\partial \mu} & \frac{\partial y}{\partial \alpha} \end{vmatrix} = \begin{vmatrix} \cos \alpha & -\mu \sin \alpha \\ \sin \theta & \mu \cos \alpha \end{vmatrix} = \mu \cos^2 \alpha + \mu \sin^2 \alpha = \boxed{\mu}$$

6. Here is a sketch of the original region and the transformed region. Transformation work is below.



Here is the work for transforming each of the sides.

$$y = -x, \quad -2 \leq x \leq 0, \quad 0 \leq y \leq 2$$

$$\sqrt{2-u} = -3v \quad \Rightarrow \quad v = -\frac{1}{3}\sqrt{2-u}$$

$$0 \leq y \leq 2 \rightarrow 0 \leq \sqrt{2-u} \leq 2 \rightarrow -2 \leq u \leq 2$$

$$-2 \leq x \leq 0 \rightarrow -2 \leq 3v \leq 0 \rightarrow -\frac{2}{3} \leq v \leq 0$$

$$y = \frac{1}{3}x, \quad 0 \leq x \leq 6, \quad 0 \leq y \leq 2$$

$$\sqrt{2-u} = \frac{1}{3}(3v) \Rightarrow v = \sqrt{2-u}$$

$$0 \leq y \leq 2 \rightarrow 0 \leq \sqrt{2-u} \leq 2 \rightarrow -2 \leq u \leq 2$$

$$0 \leq x \leq 6 \rightarrow 0 \leq 3v \leq 6 \rightarrow 0 \leq v \leq 2$$

$$y = 2, \quad -2 \leq x \leq 6$$

$$2 = \sqrt{2-u} \Rightarrow u = -2 \quad -2 \leq x \leq 6 \rightarrow -2 \leq 3v \leq 6 \rightarrow -\frac{2}{3} \leq v \leq 2$$

8. Not much to do here other than solve the transforms for x and y and plug them into the equation.

$$\left(\frac{u}{a}\right)^2 + \left(\frac{v}{b}\right)^2 \leq 1 \quad \Rightarrow \quad \frac{u^2}{a^2} + \frac{v^2}{b^2} \leq 1$$

The new region is then an ellipse with vertices $(-a, 0)$, $(a, 0)$, $(0, b)$, $(0, -b)$.

10. We saw in #8 how to turn a disk into an ellipse and so it shouldn't be too surprising that the transformation is : $x = u$, $y = 2v$. Using this transformation we get,

$$u^2 + \frac{(2v)^2}{4} = u^2 + v^2 = 1$$

The Jacobian is,

$$\begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = 2$$

The integral is then,

$$\iint_R xy \, dA = \iint_S (u)(2v) |2| \, dA = \int_0^{2\pi} \int_0^1 4r^3 \cos \theta \sin \theta \, dr \, d\theta = \int_0^{2\pi} \frac{1}{2} \sin(2\theta) \, d\theta = \boxed{0}$$

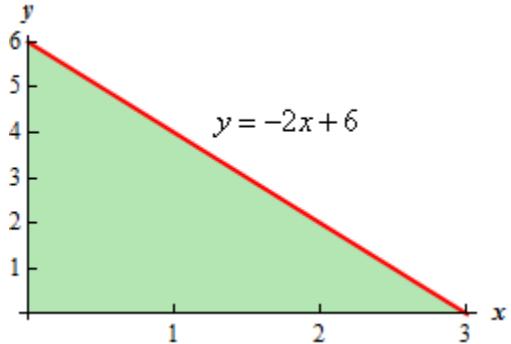
region is then an ellipse with vertices $(-a, 0), (a, 0), (0, b), (0, -b)$.

11. A sketch of the region D is to the right and the limits here are,

$$0 \leq x \leq 3 \quad 0 \leq y \leq 6 - 2x$$

The function is $z = 8 - \frac{8}{3}x - \frac{4}{3}y$ and the surface area is,

$$\begin{aligned} A &= \iint_D \sqrt{\frac{64}{9} + \frac{16}{9} + 1} dA = \frac{\sqrt{89}}{9} \int_0^3 \int_0^{6-2x} dy dx \\ &= \frac{\sqrt{89}}{5} \int_0^3 6 - 2x dx = \boxed{\sqrt{89}} \end{aligned}$$



12. The region D will be the intersection of the two surfaces.

$$11 = 4x^2 + 4y^2 - 1 \quad \Rightarrow \quad x^2 + y^2 = 3$$

The surface area is,

$$S = \iint_D \sqrt{1+16x^2+16y^2} dA = \int_0^{2\pi} \int_0^{\sqrt{3}} r \sqrt{1+16r^2} dr d\theta = \int_0^{2\pi} \frac{1}{48} (49^{\frac{3}{2}} - 1) d\theta = \boxed{\frac{57\pi}{4}}$$

13. The formula we'll need to use here for $y = f(x, z)$ is,

$$S = \iint_D \sqrt{[f_x]^2 + 1 + [f_z]^2} dA$$

A sketch of the region D is to the right. For reasons that will be apparent once we do the integral we'll use the following limits

$$0 \leq z \leq 3 \quad 0 \leq x \leq 2z$$

The area is then,

$$\begin{aligned} A &= \iint_D \sqrt{50+16z^2} dA = \int_0^3 \int_0^{2z} \sqrt{50+16z^2} dx dz \\ &= \int_0^3 2z \sqrt{50+16z^2} dz = \boxed{\frac{1}{24} (194^{\frac{3}{2}} - 50^{\frac{3}{2}})} \end{aligned}$$

