

**1. (2 pts)**  $\boxed{\nabla f = \left\langle 2x \ln\left(\frac{y^3}{x^2}\right) - 2x, \frac{3x^2}{y} \right\rangle}$

**6. (2 pts)** This is going “backwards” in  $y$  so we need to use the vector form starting at  $(-3, 13)$  and ending at  $(-3, -2)$

$$\boxed{\vec{r}(t) = (1-t)\langle -3, 13 \rangle + t\langle -3, -2 \rangle = \langle -3, 13 - 15t \rangle \quad 0 \leq t \leq 1}$$

**8. (2 pts)** The parametric equations for the curve, with counter-clockwise orientation, could be either of the following to pick two of the easiest.

$$\vec{r}(t) = \langle 2 \cos t, -2 \sin t \rangle, \pi \leq t \leq \frac{3\pi}{2} \quad \vec{r}(t) = \langle -2 \cos t, 2 \sin t \rangle, 0 \leq t \leq \frac{\pi}{2}$$

I'll use the one simply because of fewer minus signs. The integral is,

$$\int_C 4x + 7y \, ds = \int_{\pi}^{\frac{3\pi}{2}} (8 \cos t - 14 \sin t) \sqrt{(-2 \cos t)^2 + (-2 \sin t)^2} \, dt = 2 \int_{\pi}^{\frac{3\pi}{2}} 8 \cos t - 14 \sin t \, dt = \boxed{12}$$

**9. (2 pts)** Not much to do here other than do the integral.

$$\int_C \frac{(x-1)^3 y^2}{z} \, ds = \int_1^4 \frac{(1-t-1)^3 (2t)^2}{3t} \sqrt{(-1)^2 + (2)^2 + (3)^2} \, dt = -\frac{4\sqrt{14}}{3} \int_1^4 t^4 \, dt = \boxed{-\frac{1364\sqrt{14}}{5}}$$

**11. (2 pts)** Here are the parametric equations for each curve.

$$C_1 : \vec{r}(t) = \langle -4 + 4t, 4t \rangle, 0 \leq t \leq 1$$

$$C_2 : \vec{r}(t) = \langle 4 \sin t, 4 \cos t \rangle, 0 \leq t \leq \pi$$

Here is the integral for each curve.

$$\int_{C_1} x^3 dx - (x^2 - 1) dy = \int_0^1 (4t - 4)^3 (4) - ((4t - 4)^2 - 1)(4) \, dt = \left( \frac{1}{4} (4t - 4)^4 - \frac{1}{3} (4t - 4)^3 + 4t \right) \Big|_0^1 = \boxed{-\frac{244}{3}}$$

$$\int_{C_2} x^3 dx - (x^2 - 1) dy = \int_0^\pi 64 \sin^3 t (4 \cos t) - (16 \sin^2 t - 1)(-4 \sin t) \, dt$$

$$= \int_0^\pi 256 \sin^3 t \cos t + 4(15 - 16 \cos^2 t) \sin t \, dt$$

$$= \left( 64 \sin^4 t - 4(15 \cos t - \frac{16}{3} \cos^3 t) \right) \Big|_0^\pi = \boxed{\frac{280}{3}}$$

$$\int_C x^3 dx - (x^2 - 1) dy = \int_{C_1} x^3 dx - (x^2 - 1) dy + \int_{C_2} x^3 dx - (x^2 - 1) dy = \boxed{12}$$

***Not Graded***

2.  $\nabla f = \left\langle \frac{4}{y+2z}, -\frac{4x-3z}{(y+2z)^2}, \frac{-8x-3y}{(y+2z)^2} \right\rangle$

3.  $\vec{r}(t) = \langle \frac{1}{2} \cos t, 10 \sin t \rangle \quad \text{or} \quad x = \frac{1}{2} \cos t, y = 10 \sin t \quad 0 \leq t \leq 2\pi$

4.  $\vec{r}(t) = \langle t, t^3 - e^{-6t} \rangle \quad \text{or} \quad x = t, y = t^3 - e^{-6t} \quad -3 \leq t \leq 0$

5.  $\vec{r}(t) = (1-t)\langle 4, -1 \rangle + t\langle 1, 23 \rangle = \langle 4-3t, -1+24t \rangle \quad 0 \leq t \leq 1$

7.

(a) First we'll need the vector function for the line :

$$\vec{r}(t) = (1-t)\langle -1, 1 \rangle + t\langle -2, -6 \rangle = \langle -1-t, 1-7t \rangle, \quad 0 \leq t \leq 1$$

The integral is then,

$$\int_C 12x^3 ds = \int_0^1 12(-1-t)^3 \sqrt{(-1)^2 + (-7)^2} dt = 12\sqrt{50} \int_0^1 (-1-t)^3 dt = \boxed{-225\sqrt{2}}$$

(b) Here are the parametric equations for each portion of the curve and following the given orientation as instructed to do by the problem statement.

$$C_1 : \vec{r}(t) = \langle -1, t \rangle, \quad 1 \leq t \leq 3 \quad (\text{line } x = -1)$$

$$C_2 : \vec{r}(t) = \langle t, 2-t^3 \rangle, \quad -1 \leq t \leq 2 \quad (\text{curve } y = 2-x^3)$$

$$C_3 : \vec{r}(t) = \langle 2-4t, -6 \rangle, \quad 0 \leq t \leq 1 \quad (\text{line } y = 6)$$

Here's the integral for each curve.

$$\int_{C_1} 12x^3 ds = \int_1^3 12(-1)^3 \sqrt{(0)^2 + (1)^2} dt = \int_1^3 -12 dt = \boxed{-24}$$

$$\int_{C_2} 12x^3 ds = \int_{-1}^2 12(t)^3 \sqrt{(1)^2 + (-3t^2)^2} dt = \int_{-1}^2 12t^3 \sqrt{1+9t^4} dt = \frac{2}{9} \left[ 145^{\frac{3}{2}} - 10^{\frac{3}{2}} \right]$$

$$\int_{C_3} 12x^3 ds = \int_0^1 12(2-4t)^3 \sqrt{(-4)^2 + (0)^2} dt = \int_0^1 48(2-4t)^3 dt = \boxed{0}$$

The overall integral is then,

$$\int_C 12x^3 \, ds = \int_{C_1} 12x^3 \, ds + \int_{C_2} 12x^3 \, ds + \int_{C_3} 12x^3 \, ds = \boxed{\frac{2}{9} \left( 145^{\frac{3}{2}} - 10^{\frac{3}{2}} \right) - 24 = 356.98}$$

**10.** Not much to do here other than do the integral.

$$\begin{aligned} \int_C \cos(4y) \, dy &= \int_0^{\ln(2)} \cos(4e^{2t}) [2e^{2t}] \, dt = \frac{1}{4} \sin(4e^{2t}) \Big|_0^{\ln(2)} \\ &= \frac{1}{4} (\sin(4e^{2\ln(2)}) - \sin(4)) \quad e^{2\ln(2)} = e^{\ln(2^2)} = 4 \\ &= \boxed{\frac{1}{4} (\sin(16) - \sin(4)) = 0.1172} \end{aligned}$$

**12.** Here is the parametric equation for the line.

$$\vec{r}(t) = \langle 2 - 3t, 4t - 3, 2t \rangle \quad 0 \leq t \leq 1$$

The integral is then,

$$\begin{aligned} \int_C y^3 \, dx - (2 - 4z) \, dy + 2xz \, dz &= \int_0^1 (4t - 3)^3 (-3) - (2 - 4(2t))(4) + 2(2 - 3t)(2t)(2) \, dt \\ &= \int_0^1 -3(4t - 3)^3 - 24t^2 + 48t - 8 \, dt = \boxed{23} \end{aligned}$$