Math 2415 **Homework Set 9** **10 Points**

Line Integrals of Vector Fields

For problems 1 and 2 evaluate $\int_C \vec{F} \bullet d\vec{r}$ for the given vector field, $\vec{F}$, and the given curve, *C*.

1. $\vec{F}(x,y) = xy\,\vec{i} - (1-y)\,\vec{j}$ where *C* is the left half of $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and with counter clockwise motion.

2. $\vec{F}(x,y,z) = yz\,\vec{i} - \left(x+z^2\right)\vec{j} + \cos(4y)\,\vec{k}$ and *C* is given by $\vec{r}(t) = (9-t)\,\vec{i} + t^3\,\vec{j} - 6t^3\,\vec{k}$, $0 \le t \le 1$

Fundamental Theorem for Line Integrals

3. Evaluate $\int_C \nabla f \bullet d\vec{r}$ for $f(x,y,z) = x^2\sin\left(5x^3 - y\right) + 2xyz$ and *C* is given by $\vec{r}(t) = \left\langle t^2 + t, 2t^3 + 6, 7t^2 \right\rangle$, $-1 \le t \le 3$.

4. For the vector field in **#1** above is it possible to determine if $\int_C \vec{F} \bullet d\vec{r}$ is independent of path? If so, is the integral independent of path and if it is not possible to determine explain why not.

Conservative Vector Fields

For problems 5 and 6 determine if the vector field, $\vec{F}$, is conservative or not. If it is conservative find the potential function for the vector field.

5. $\vec{F} = \left(8xy^3 + 3x^2y\right)\vec{i} + \left(12x^2y^2 - x^3\right)\vec{j}$

6. $\vec{F} = \left(6x + 3x^2y^4\right)\vec{i} - \left(8y - 4x^3y^3 + 7\right)\vec{j}$

For problems 7 – 9 find the potential function for the vector field and then evaluate $\int_C \vec{F} \bullet d\vec{r}$ for the given curve *C*.

7. $\vec{F} = \left(6x + 3x^2y^4\right)\vec{i} - \left(8y - 4x^3y^3 + 7\right)\vec{j}$ and *C* is the line segment from (1, -3) to (4, 0).

8. $\vec{F} = \left(-4x - \mathbf{e}^{2y}\cos(4y-x)\right)\vec{i} + \left(1 + 2\mathbf{e}^{2y}\sin(4y-x) + 4\mathbf{e}^{2y}\cos(4y-x)\right)\vec{j}$ and *C* is the bottom half of the circle of radius 4 that is centered at (-1, 2) starting at the right and ending at the left.

9. $\vec{F} = 2x\ln\left(y^2z\right)\vec{i} - \left(27y^2z^4 - \frac{2x^2}{y}\right)\vec{j} + \left(\frac{x^2}{z} - 36y^3z^3\right)\vec{k}$ and *C* is $\vec{r}(t) = (4t-8)\,\vec{i} + 2t^2\,\vec{j} + \vec{k}$, $1 \le t \le 2$.

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<u>Green's Theorem</u>

For problems 10 and 11 evaluate the line integral (a) directly and (b) using Green's Theorem. Assume all curves have the positive orientation.

10. $\oint_C (2xy+3y)\,dx-(3x-x^2)\,dy$ where *C* is the circle of radius 3 centered at the origin with positive orientation.

11. $\oint_C x^2\,dy-(4y-8xy)\,dx$ where *C* is the triangle with vertices (0,0), (3,9) and (3,-1) with positive orientation.