Math 2415 **Homework Set 10** **10 Points**

Green's Theorem

For problems 1 and 2 sketch the positively oriented curve (clearly indicating the positive orientation) and use Green's Theorem to evaluate the line integral along the given curve.

1. $\oint\limits_C 5(1-4xy)\,dx + 9x\,dy$ where *C* is the portion of $x = (y-3)^2$ in the range $1 \le y \le 5$ and the three line segments with endpoints : (*i*) (4,1) & (7,1), (*ii*) (7,1) & (7,5), (*iii*) (4,5) & (7,5) .

2. $\int\limits_C \vec{F} \bullet d\vec{r}$ where $\vec{F} = \left(xy^2 - 2y^3\right)\vec{i} + 2x^3\vec{j}$ and *C* is the shorter portion of $x^2 + y^2 = 9$ between $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$ & $\left(-\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$ and the two line segments with endpoints : (i) from $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$ & (0,0) and (*ii*) $\left(-\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$ & (0,0).

Curl and Diverence

3. Find the curl and divergence of $\vec{F} = x^2y^3z^4\vec{i} + x\ln(z)\,\vec{j} + (10z - 9y - 8x)\vec{k}$

For problems 4 and 5 use the curl to determine if the given vector field is conservative or not.

4. $\vec{F} = x^2y^3z^4\vec{i} + x\ln(z)\,\vec{j} + (10z - 9y - 8x)\vec{k}$

5. $\vec{F} = 2x\ln\left(y^2z\right)\vec{i} - \left(27y^2z^4 - \frac{2x^2}{y}\right)\vec{j} + \left(\frac{x^2}{z} - 36y^3z^3\right)\vec{k}$

Parametric Surfaces

For problems 6 – 9 find a parametric representation for the given surface.

6. The plane containing the points (-1,2,4), (-1,0,3) and (5,2,3).

7. The portion of $y = 7x^2 + 7z^2 - 9$ that lies behind $y = 10$.

8. The cylinder $y^2 + z^2 = 15$ between $x = 20$ and $x = 30$.

9. The portion of the sphere $x^2 + y^2 + z^2 = 100$ with $x \ge 0$ and $y \le 0$.

10. Find the tangent plane to $x = u^2 + 4u - 17$, $y = u^2 + uv^3$, $z = 5v^2$ at the point (15,128,20).

For problems 11 and 12 find the area of the given surface.

11. The portion of $z = 5 - 2x^2 - 2y^2$ that lies above the plane given by $z = 1$.

12. The surface $r(u,v) = u^2\,\vec{i} + (3u - v)\,\vec{j} + (1 + 2v)\vec{k}$ where *u* and *v* are in the triangle with vertices given by (0,0), (8,0) and (8,4) – these are in the form (u,v).