Math 2415 **Homework Set 11** **10 Points**

Surface Integrals

For problems 1 – 3 evaluate the surface integral.

1. $\iint\limits_S 8z - 2x\,dS$ where *S* is the portion of the plane $4x + 6y + z = 12$ that lies in the first octant.

2. $\iint\limits_S 2x^2 + 2z^2 - y\,dS$ where *S* is the portion of $y = 2x^2 + 2z^2 - 8$ that lies in behind of $y = 0$.

3. $\iint\limits_S x - 3\,dS$ where *S* is the portion of the cylinder $y^2 + z^2 = 1$ and bounded by $x = -1$ and $x - z = 3$.

Surface Integrals of Vector Fields

For problems 4 and 5 evaluate $\iint\limits_S \vec{F} \cdot d\vec{S}$ for the given vector field and surface.

4. $\vec{F}(x, y, z) = z\,\vec{i} + y^2\,\vec{j} - x\,\vec{k}$ and *S* is the part of the $y = 2x^2 + 2z^2 - 9$ that lies in behind $y = -1$ and oriented in the direction of the negative *y*-axis.

5. $\vec{F}(x, y, z) = x\vec{i} - z\vec{k}$ and *S* is the surface from problem **#3** with the positive orientation.

Stokes' Theorem

6. Use Stokes' Theorem to evaluate $\iint\limits_S \operatorname{curl}\vec{F} \cdot d\vec{S}$ where $\vec{F} = y^2\,\vec{i} - y\,\vec{j} + (2x - 8z)\,\vec{k}$ and *S* is the part of the sphere $x^2 + y^2 + z^2 = 20$ that lies above the *xy*-plane and inside the cylinder $x^2 + y^2 = 8$, oriented in the direction of the positive *z*-axis.

7. Use Stokes' Theorem to evaluate $\int\limits_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = y\,\vec{i} - 2x\,\vec{j} + z^2\vec{k}$ and *C* is the circle $x^2 + y^2 = 4$ at $y = 1$ and *C* is oriented in the clockwise direction when viewed from the front (*i.e.* looking towards the negative *y* axis).

Hint : You'll need an easy to work with surface whose intersection with the plane $y = 1$ is the circle $x^2 + y^2 = 4$ that will also have the correct orientation. By this point in the semester you've worked many times with one particular kind of surface that will do this.

Divergence Theorem

8. Use the Divergence Theorem to evaluate $\iint\limits_S \vec{F} \cdot d\vec{r}$ where $\vec{F} = 3xz^2\,\vec{i} + 8xy\,\vec{j} - z^3\,\vec{k}$ and *S* is the portion of the sphere $x^2 + y^2 + z^2 = 9$ in the first octant.