1. (3 pts)

$$\int \frac{dy}{(3+y)^2} = \int \frac{dx}{1+x} \quad \to \quad -\frac{1}{3+y} = \ln|1+x| + c \quad \to \quad -\frac{1}{3+12} = -\frac{1}{15} = \ln|1+0| + c = c$$

$$\tfrac{1}{3+y} = \tfrac{1}{15} - \ln|1+x| \quad \to \quad 3+y = \tfrac{15}{1-15\ln|1+x|} \quad \to \quad \boxed{y(x) = \frac{15}{1-15\ln|1+x|} - 3}$$

For the interval of validity we can see that we'll need to require $x \neq -1$ because of the logarithm. Also we'll need to avoid division by zero so let's solve,

$$1 - 15\ln|1+x| = 0 \;\to\; \ln|1+x| = \tfrac{1}{15} \quad \to \quad |1+x| = \mathbf{e}^{\frac{1}{15}} \;\to\; 1+x = \pm\mathbf{e}^{\frac{1}{15}}$$

$$x = -1 \pm \mathbf{e}^{\frac{1}{15}} = -2.0689, 0.0689$$

The possible intervals of validity are below and the third one is the actual interval of validity.

$$-\infty < x < -2.0689 \qquad -2.0689 < x < -1 \qquad \boxed{-1 < x < 0.0689} \qquad 0.0689 < x < \infty$$

2. (2 pts)

$$\int \frac{dy}{y} = \int 5x^2 - 7x\,dx \quad \to \quad \ln y = \tfrac{5}{3}x^3 - \tfrac{7}{2}x^2 + c \quad \to \quad \ln\left(\mathbf{e}^9\right) = 9 = 0 + c \quad \to \quad c = 9$$

$$\boxed{y = \mathbf{e}^{\frac{5}{3}x^3 - \frac{7}{2}x^2 + 9}}$$

In this case we can plug in every *x* and so the interval of validity is : $\boxed{-\infty < x < \infty}$.

5. (5 pts) Here are the two IVP's for this situation.

$$Q_1' = (5)(4) - \frac{6Q_1}{600-2t} = 20 - \frac{3Q_1}{300-t} \qquad\qquad Q_1(0) = 150$$

$$Q_2' = (2)(6) - \frac{6Q_2}{V_c} = 12 - \frac{6Q_2}{V_c} \qquad\qquad Q_2(t_c) = 600$$

Note that because we don't know when the change will happen we can't completely write down the second IVP. The first IVP is a simple linear differential equation so I'll leave it to you to verify the solution.

$$Q_1(t) = 10(300-t) - \frac{2850(300-t)^3}{300^3}$$

Setting this equal to 600 and using a calculator to solve gives three solutions : 28.3106, 237.4121, and 634.2773. The first one is the one we need so $t_c = 28.3106$ and $V_c = 600 - 2(28.3106) = 543.3788$

The second IVP is then,

$$Q_2' = 12 - \frac{6Q_2}{543.3788} \qquad\qquad Q_2(28.3106) = 600$$

The second IVP is a simple linear differential equation and so I'll again leave it to you to verify the solution.

$$Q_2(t) = 1086.7576 - 665.3894\mathbf{e}^{-0.01104t} \qquad Q_2(28.3106+50) = 806.51232$$

There is 806.51232 grams in the tank 50 hours after the change (*i.e.* when $t = 78.3106$).

Not Graded

3.

$$\int 7+4y\,dy = \int 4-2x\,dx \quad \to \quad 7y+2y^2 = 4x-x^2+c \quad \to \quad 7(-6)+2(36) = 30 = 4+c$$

$$2y^2+7y-\left(4x-x^2+30\right)=0 \quad y(x) = \frac{-7\pm\sqrt{49+(4)(2)\left(4x-x^2+26\right)}}{4} = \frac{-7\pm\sqrt{257+32x-8x^2}}{4}$$

Reapplying the initial condition shows us that "-" is the correct sign. So the solution is,

$$\boxed{y(x) = \frac{-7-\sqrt{257+32x-8x^2}}{4}}$$

For the interval of validity we need to know where the quadratic under the radical is zero.

$$257+32x-8x^2 = 0 \qquad \to \qquad x = \tfrac{-32\pm\sqrt{9248}}{-16} = \tfrac{8\pm17\sqrt{2}}{4} = -4.010, 8.010$$

A quick check of signs shows us that the interval of validity must be : $\boxed{-4.010 < x < 8.010}$

Now, for the minimum value(s) of the solution. We know from Calc I that extrema must occur at critical points and end points. We know the two endpoints of the interval (above of course…) and from the differential equation we can see that the only critical points where the derivative is zero will occur at $x = 2$. From our knowledge of derivative of roots we know that if we were to differentiate the solution we'd get the root in the denominator and so we'll have critical points where that is zero, and we found those above as well as they are the endpoints of the interval. So, all we need to do then is plug in the endpoints and the critical point and identify the minimum values.

$$y(-4.010) = -1.7995 \qquad y(2) = -6 \qquad y(8.010) = -1.7995$$

So, the minimum value is -6.

4. Here is the IVP for this situation.

$$Q' = 4\left(5+10\,\mathbf{e}^{-\frac{t}{200}}\right) - \frac{4Q}{400} = 20+40\,\mathbf{e}^{-\frac{t}{200}} - \frac{Q}{100} \qquad Q(0) = 60$$

This is a fairly simple linear differential equation and so I'll leave the solution details to you.

$$Q(t) = 2000 - 9940\mathbf{e}^{-\frac{t}{100}} + c\mathbf{e}^{-\frac{t}{200}} = 2000 - 9940\mathbf{e}^{-\frac{t}{100}} + 8000\mathbf{e}^{-\frac{t}{200}}$$

Taking the limit of this as $t \to \infty$ we can see that the equilibrium amount of salt will be 2000 ounces.

6. Here's the IVP for this situation.

$$Q_3' = (0)(10) - \frac{8Q_3}{543.3788 + 2(t - 78.3106)} \qquad Q_2(78.3106) = 806.51232$$