Math 3301 **Homework Set 3 – Solutions** **10 Points**

1. (3 pts) Here's the IVP's we need for this problem. Note that I'm using a time frame of months here and so all per week quantities will need to be multiplied by 4 to get them into a per month quantity.

$$P' = rP \qquad P(0) = 300 \qquad P(2) = 1200$$

$$P' = rP + 30(4) - 40(4) = rP - 40 \qquad P(0) = 300$$

Solving the first and applying the initial condition gives the following solution which we can then apply the second condition,

$$P(t) = 300\mathbf{e}^{rt} \qquad 1200 = 300\mathbf{e}^{2r} \qquad r = \tfrac{1}{2}\ln(4) = \ln(4)^{\frac{1}{2}} = \ln(2)$$

The second IVP is now,

$$P' = \ln(2)P - 40 \qquad P(0) = 300$$

I'll leave it to you to verify that the solution is,

$$P(t) = \tfrac{40}{\ln 2} + 242.2922\mathbf{e}^{\ln(2)t}$$

From this we can see that the insects will survive because everything is positive and the exponential will go to infinity as $t \to \infty$.

3. (3 pts) Here's the IVP for this case.

$$v' = 9.8 - \tfrac{30}{20}v = 9.8 - \tfrac{3}{2}v \qquad v(0) = 0.75$$

I'll leave it to you to verify the solution to this.

$$v(t) = 6.5333 - 5.7833\mathbf{e}^{-\frac{3}{2}t}$$

To determine when it hits the ground we can set this equal to 5 and solve for *t*.

$$5 = 6.5333 - 5.7833\mathbf{e}^{-\frac{3}{2}t} \quad \to \quad 0.26513 = \mathbf{e}^{-\frac{3}{2}t} \quad \Rightarrow \quad t = 0.8850$$

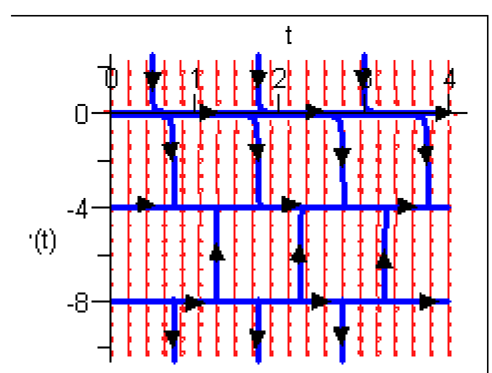
The height function is,

$$s(t) = \int 6.5333 - 5.7833\mathbf{e}^{-\frac{3}{2}t}\,dt \qquad s(0) = 0 \quad \Rightarrow \quad s(t) = 6.5333t + 3.8556\mathbf{e}^{-\frac{3}{2}t} - 3.8556$$

The bridge is then $s(0.8850) = 2.9487\,m$ above the ground.

4. (2 pts) The equilibrium solutions are : $y = -8$, $y = -4$, and $y = 0$. From a sketch of the solutions we can see the following classifications.

$$y = 0 \ : \ \text{Semi-stable}$$
$$y = -4 \ : \ \text{Asymp. Stable}$$
$$y = -8 \ : \ \text{Unstable}$$



6. (2 pts) We just need to run through the formulas using $f(t,y) = y + t^2 - \sin(y)$. Here's the results for $h = 0.4$.

t	2.4	2.8
f_n	10.3430134013	17.8871924346
Approx.	11.1372053605	18.2920823344

Here's the results for $h = 0.2$.

t	2.2	2.4	2.6	2.8
f_n	10.3430134013	13.5599105577	18.2479656363	21.9159513489
Approx.	9.0686026803	11.7805847918	15.4301779190	19.8133681888

For $h = 0.4$ we have $y(2.8) \approx 18.2920823344$ and for $h = 0.2$ we have $y(2.8) \approx 19.8133681888$.

Not Graded

2. The new IVP that we'll need for this new situation is,

$$P_2' = \ln(2)P_2 + 30(4) - 40(4) - 120(4) = \ln(2)P_2 - 520 \qquad P_2(1.5) = P_1(1.5) = 743.0136$$

I'll let you verify that the solution to this IVP is,

$$P(t) = \tfrac{520}{\ln 2} - 2.5413\mathbf{e}^{\ln(2)t}$$

So, it looks like the insects will now die out (although just barely is seems as *c* is only just switched over to negative!). Setting equal to zero and solving gives that they will die at $t = 8.2056$. So, they will die out after about 33 weeks (or so….).

5. The equilibrium solutions are : $y = 0$ and $y = 2$. From a sketch of the solutions we can see the following classifications.

$$y = 2 \ : \ \text{Asymp. Stable}$$
$$y = 0 \ : \ \text{Unstable}$$