

4. (4 pts) $r^2 - 7r - 18 = (r+2)(r-9) = 0 \rightarrow r_1 = -2, r_2 = 9 \rightarrow y(t) = c_1 e^{-2t} + c_2 e^{9t}$

$$\begin{aligned} c_1 + c_2 &= 3 + \beta & c_1 &= \frac{1}{11}(26 + 9\beta + \beta^2) \\ -2c_1 + 9c_2 &= 1 - \beta^2 & c_2 &= \frac{1}{11}(7 + 2\beta - \beta^2) \\ y(t) &= \frac{1}{11}(26 + 9\beta + \beta^2)e^{-2t} + \frac{1}{11}(7 + 2\beta - \beta^2)e^{9t} \end{aligned}$$

The first term will go to zero at $t \rightarrow \infty$ and the exponential in the second will go to ∞ at $t \rightarrow \infty$. So, in order for the solution to remain finite the coefficient of the second term must be zero or,

$$7 + 2\beta - \beta^2 = 0 \rightarrow \boxed{\beta = \frac{-2 \pm \sqrt{32}}{-2} = 1 \pm 2\sqrt{2}}$$

6. (3 pts) $r^2 + 2r + 50 = 0 \rightarrow r_{1,2} = -1 \pm 7i \rightarrow y(t) = c_1 e^{-t} \cos(7t) + c_2 e^{-t} \sin(7t)$

$$\begin{aligned} c_1 &= 9 & c_1 &= 9 \\ -c_1 + 7c_2 &= -2 & c_2 &= 1 \end{aligned} \rightarrow \boxed{y(t) = 9e^{-t} \cos(7t) + e^{-t} \sin(7t)}$$

7. (3 pts) $r^2 + 2\sqrt{3}r + 3 = (r + \sqrt{3})^2 = 0 \rightarrow r_{1,2} = -\sqrt{3} \rightarrow y(t) = c_1 e^{-\sqrt{3}t} + c_2 t e^{-\sqrt{3}t}$

$$\begin{aligned} c_1 &= -7 & c_1 &= -7 \\ -\sqrt{3}c_1 + c_2 &= 0 & c_2 &= -7\sqrt{3} \end{aligned} \rightarrow \boxed{y(t) = -7e^{-\sqrt{3}t} - 7\sqrt{3}t e^{-\sqrt{3}t}}$$

Not Graded

1. $5r^2 - r - 3 = 0 \rightarrow r_{1,2} = \frac{1 \pm \sqrt{61}}{10} \rightarrow \boxed{y(t) = c_1 e^{\frac{1+\sqrt{61}}{10}t} + c_2 e^{\frac{1-\sqrt{61}}{10}t}}$

2. $6r^2 + 11r + 3 = (2r+3)(3r+1) = 0 \rightarrow r_1 = -\frac{3}{2}, r_2 = -\frac{1}{3} \rightarrow y(t) = c_1 e^{-\frac{3}{2}t} + c_2 e^{-\frac{1}{3}t}$

$$\begin{aligned} c_1 + c_2 &= 0 & c_1 &= \frac{6}{7} \\ -\frac{3}{2}c_1 - \frac{1}{3}c_2 &= -1 & c_2 &= -\frac{6}{7} \end{aligned} \rightarrow \boxed{y(t) = \frac{6}{7}e^{-\frac{3}{2}t} - \frac{6}{7}e^{-\frac{1}{3}t}}$$

3. $r^2 - 16 = 0 \rightarrow r_{1,2} = \pm 4 \rightarrow y(t) = c_1 e^{-4t} + c_2 e^{4t}$

$$\begin{aligned} c_1 + c_2 &= 2 & c_1 &= \frac{11}{4} \\ -4c_1 + 4c_2 &= -14 & c_2 &= -\frac{3}{4} \end{aligned} \rightarrow \boxed{y(t) = \frac{11}{4}e^{-4t} - \frac{3}{4}e^{4t}}$$

5. $4r^2 - 24r + 37 = 0 \rightarrow r_{1,2} = 3 \pm \frac{1}{2}i \rightarrow y(t) = c_1 e^{3t} \cos\left(\frac{t}{2}\right) + c_2 e^{3t} \sin\left(\frac{t}{2}\right)$

$$\begin{aligned} -c_1 e^{6\pi} &= 0 & c_1 &= 0 \\ -3c_1 e^{6\pi} - \frac{1}{2}c_2 e^{6\pi} &= -1 & c_2 &= 2e^{-6\pi} \end{aligned} \rightarrow \boxed{y(t) = 2e^{3t-6\pi} \sin\left(\frac{t}{2}\right)}$$

8. $16r^2 - 24r + 9 = (4r - 3)^2 = 0 \rightarrow r_{1,2} = \frac{3}{4} \rightarrow y(t) = c_1 e^{\frac{3}{4}t} + c_2 t e^{\frac{3}{4}t}$

$$\begin{aligned} c_1 e^{\frac{3}{4}} + c_2 e^{\frac{3}{4}} &= 1 & \rightarrow c_1 = -\frac{1}{4} e^{-\frac{3}{4}} & \rightarrow y(t) = -\frac{1}{4} e^{\frac{3}{4}t - \frac{3}{4}} + \frac{5}{4} t e^{\frac{3}{4}t - \frac{3}{4}} \\ \frac{3}{4} c_1 e^{\frac{3}{4}} + \frac{7}{4} c_2 e^{\frac{3}{4}} &= 2 & c_2 = \frac{5}{4} e^{-\frac{3}{4}} & = -\frac{1}{4} e^{\frac{3}{4}(t-1)} + \frac{5}{4} t e^{\frac{3}{4}(t-1)} \end{aligned}$$