

Undetermined Coefficients, Part II

For problems 1 & 2 use the method of undetermined coefficients to determine the general solution to the given differential equation.

1. $y'' - 7y' = 5t - 1 + \cos(2t)$

2. $y'' - 7y' = 6e^{-2t} - 3e^{7t}$

3. Solve the following IVP using the method of undetermined coefficients.

$$y'' - 4y' + 4y = 7e^{2t} \qquad y(0) = 12, \quad y'(0) = -1$$

For problems 4 & 5 write down the guess that we'd need to use with the method of undetermined coefficients to find the particular solution. Do not attempt to find the actual particular solution.

4. $y'' - 8y' + 25y = 3t \cos(3t) - t^2 e^{4t} + 5e^{4t} \sin(3t)$

5. $y'' + y = e^{9t} - (2 + 7t) \cos(t) + t^2 \sin(t) - 4te^{9t}$

Variation of Parameters

6. Use the method of variation of parameters to find the solution to the following differential equation.

$$y'' - 2y' - 8y = e^{4t} - 3e^{-t}$$

7. Use the method of variation of parameters to find the solution to the following IVP.

$$9y'' + y = 10 \qquad y(0) = -8, \quad y'(0) = 7$$

Vibrations

For problems 8 – 11 any solutions containing both a sine and a cosine must be combined into a single cosine. Any decimal work should be to at least the 4th decimal place.

8. A 18 lb object will stretch a spring 8 inches by itself. The mass has no damping and is initially displaced 2 inches upwards from its equilibrium position with an initial velocity of 8 in/sec upwards. Determine the displacement at any time t .

9. A 800 gram object will stretch a spring 25 cm by itself. The mass has a damper hooked up that will exert a force of 6N when the velocity is 30 cm/sec. The mass is initially released from its equilibrium position with an initial velocity of 10 cm/sec downwards. Determine the displacement at any time t . What kind of damping does the system experience?

10. Take the system from #8 and hook up a forcing function of the form $g(t) = 9\cos(2t) + 4\sin(2t)$ and determine the displacement at any time t . Will this system experience resonance?

11. Take the system from #9 and hook up a forcing function of the form $g(t) = 11\sin(4t)$. Determine the displacement at any time t .