

Step Functions

For problems 1 – 3 find the Laplace transform of the given function.

1. $g(t) = 3u_2(t)e^{8-4t} + 7u_6(t)\cos\left(\frac{1}{3}t - 2\right)$

2. $f(t) = u_4(t)\sin(3t) - 2t^2u_3(t)$

3.
$$h(t) = \begin{cases} -2 & t < 6 \\ 4t & 6 \leq t < 9 \\ 3 + 4t - e^t & t \geq 9 \end{cases}$$

For problems 4 – 6 find the inverse Laplace transform of each of the following

4. $F(s) = \frac{s e^{-3s}}{s^2 + 4} + \frac{9}{s} - \frac{3e^{-7s}}{s}$

5. $F(s) = \frac{6e^{-s} + s^2 e^{-8s} + 11e^{-2s}}{s(s+2)^2}$

6. $H(s) = \frac{11e^{-7s} - 9s - 2se^{-17s}}{(s-4)(s^2 + 9)}$

IVP's with Laplace Transforms

Use Laplace transforms to solve the given IVP. In the partial fraction stage all quadratics that can be factored with integer coefficients must be factored.

7. $y'' + 9y = 4\cos(3t) \quad y(0) = 0, \quad y'(0) = 0$

8. $2y'' - 5y' - 3y = 2e^{6t} \quad y(0) = -2, \quad y'(0) = 1$

9. $y'' + 2y' + 5y = 4t \quad y(0) = 0, \quad y'(0) = -6$