

2. (2 pts) In this case neither of the two functions are properly shifted and so we'll need to do some work first.

$$\begin{aligned} f(t) &= u_4(t) \sin(3(t-4+4)) - 2(t-3+3)^2 u_3(t) \\ &= u_4(t) \sin(3(t-4)+12) - 2[(t-3)^2 + 6(t-3) + 9] u_3(t) \end{aligned}$$

Note that with the second term I just used the fact that $(a+b)^2 = a^2 + 2ab + b^2$ with $a=t-3$, $b=3$.

So, in the first case we're shifting $\sin(3t+12)$ (using #15 from the table) and in the second we're shifting $t^2 + 6t + 9$. The transform is then,

$$F(s) = \boxed{\frac{e^{-4s}(s \sin(12) + 3 \cos(12))}{s^2 + 9} - 2e^{-3s}\left(\frac{2}{s^3} + \frac{6}{s^2} + \frac{9}{s}\right)}$$

5. (2 pts)

$$\begin{aligned} F(s) &= \frac{6e^{-s} + 11e^{-8s}}{s(s+2)^2} + \frac{s^2 e^{-2s}}{s(s+2)^2} = (6e^{-s} + 11e^{-8s}) \frac{1}{s(s+2)^2} + e^{-2s} \frac{s}{(s+2)^2} \\ &= (6e^{-s} + 11e^{-8s}) G(s) + e^{-2s} H(s) \end{aligned}$$

I'll leave it to you to verify the partial fraction work.

$$\begin{aligned} G(s) &= \frac{1}{4} \left[\frac{1}{s} - \frac{1}{s+2} - \frac{2}{(s+2)^2} \right] &\rightarrow g(t) &= \frac{1}{4} (1 - e^{-2t} - 2te^{-2t}) \\ H(s) &= \frac{1}{s+2} - \frac{2}{(s+2)^2} &\rightarrow h(t) &= e^{-2t} - 2te^{-2t} \end{aligned}$$

The inverse Laplace Transform is then,

$$F(s) = 6e^{-s}G(s) + 11e^{-8s}G(s) + e^{-2s}H(s) \rightarrow \boxed{f(t) = 6u_1(t)g(t-1) + 11u_8(t)g(t-8) + u_2(t)h(t-2)}$$

7. (3 pts) Take the Laplace transform of everything.

$$\begin{aligned} s^2 Y(s) - sy(0) - y'(0) + 9Y(s) &= \frac{4s}{s^2 + 9} \\ (s^2 + 9)Y(s) &= \frac{4s}{s^2 + 9} \quad \rightarrow \quad Y(s) = \frac{4s}{(s^2 + 9)^2} \end{aligned}$$

Note that we don't need to do any partial fractions here. This is simply #9 from the table.

$$Y(s) = \frac{2(2)(\frac{3}{3})s}{(s^2 + 9)^2} \quad \rightarrow \quad \boxed{y(t) = \frac{2}{3}t \sin(3t)}$$

8. (3 pts) Take the Laplace transform of everything.

$$\begin{aligned} 2(s^2Y(s) - sy(0) - y'(0)) - 5(sY(s) - y(0)) - 3Y(s) &= \frac{2}{s-6} \\ (2s^2 - 5s - 3)Y(s) + 4s - 12 &= \frac{2}{s-6} \\ (2s^2 - 5s - 3)Y(s) &= \frac{2}{s-6} + 12 - 4s = \frac{2 + (12 - 4s)(s-6)}{s-6} \\ Y(s) &= \frac{-4s^2 + 36s - 70}{(s-6)(s-3)(2s+1)} \end{aligned}$$

I'll leave it to you to verify the partial fraction work.

$$Y(s) = \frac{\frac{2}{39}}{s-6} - \frac{\frac{2}{21}}{s-3} - \frac{\frac{356}{91}}{2(s+\frac{1}{2})} \quad \rightarrow \quad \boxed{y(t) = \frac{2}{39}e^{6t} - \frac{2}{21}e^{3t} - \frac{178}{91}e^{-\frac{1}{2}t}}$$

Not Graded

1. $g(t) = 3u_2(t)e^{-4(t-2)} + 7u_6(t)\cos(\frac{1}{3}(t-6))$ $G(s) = \frac{3e^{-2s}}{s+4} + \frac{7se^{-6s}}{s^2 + \frac{1}{9}}$

3. First we'll need to write the function in terms of Heaviside functions and get things properly shifted.

$$\begin{aligned} h(t) &= -2 + (4t+2)u_6(t) + (3 - e^t)u_9(t) = -2 + (4(t-6+6)+2)u_6(t) + (3 - e^{t-9+9})u_9(t) \\ &= -2 + (4(t-6)+26)u_6(t) + (3 - e^9e^{t-9})u_9(t) \end{aligned}$$

Don't get excited about the e^9 in the second term. It is just a number and so treat it in the same manner you would a 9, for instance.

$H(s) = -\frac{2}{s} + e^{-6s} \left(\frac{4}{s^2} + \frac{26}{s} \right) + e^{-9s} \left(\frac{3}{s} - \frac{e^9}{s-1} \right)$

4. $f(t) = u_3(t)\cos(2(t-3)) + 9 - 3u_7(t)$

6.

$$H(s) = 11e^{-7s} \frac{1}{(s-4)(s^2+9)} - (9 + 2e^{-17s}) \frac{s}{(s-4)(s^2+9)} = 11e^{-7s}F(s) - (9 + 2e^{-17s})G(s)$$

I'll leave it to you to verify the partial fraction work.

$$F(s) = \frac{1}{25} \left[\frac{1}{s-4} - \frac{s+4}{s^2+9} \right] = \frac{1}{25} \left[\frac{1}{s-4} - \frac{s}{s^2+9} - \frac{4\frac{3}{3}}{s^2+9} \right] \rightarrow f(t) = \frac{1}{25} (\mathbf{e}^{4t} - \cos(3t) - \frac{4}{3} \sin(3t))$$

$$G(s) = \frac{1}{25} \left[\frac{4}{s-4} + \frac{9-4s}{s^2+9} \right] = \frac{1}{25} \left[\frac{4}{s-4} + \frac{3(3)}{s^2+9} - \frac{4s}{s^2+9} \right] \rightarrow g(t) = \frac{1}{25} (4\mathbf{e}^{4t} + 3 \sin(3t) - 4 \cos(3t))$$

The inverse Laplace Transform is then,

$$H(s) = 11\mathbf{e}^{-7s} F(s) - 9G(s) - 2\mathbf{e}^{-17s} G(s) \rightarrow h(t) = 11u_7(t)f(t-7) - 9g(t) - 2u_{17}(t)g(t-17)$$

9. Take the Laplace transform of everything.

$$s^2 Y(s) - sy(0) - y'(0) + 2(sY(s) - y(0)) + 5Y(s) = \frac{4}{s^2}$$

$$(s^2 + 2s + 5)Y(s) + 6 = \frac{4}{s^2}$$

$$(s^2 + 2s + 5)Y(s) = \frac{4}{s^2} - 6 = \frac{4 - 6s^2}{s^2}$$

$$Y(s) = \frac{4 - 6s^2}{s^2(s^2 + 2s + 5)}$$

I'll leave it to you to verify the partial fraction work.

$$Y(s) = \frac{1}{25} \left[-\frac{8}{s} + \frac{20}{s^2} + \frac{8s - 154}{(s+1)^2 + 4} \right] = \frac{1}{25} \left[-\frac{8}{s} + \frac{20}{s^2} + \frac{8(s+1-1) - 154}{(s+1)^2 + 4} \right]$$

$$= \frac{1}{25} \left[-\frac{8}{s} + \frac{20}{s^2} + \frac{8(s+1)}{(s+1)^2 + 4} - \frac{162}{(s+1)^2 + 4} \right] = \frac{1}{25} \left[-\frac{8}{s} + \frac{20}{s^2} + \frac{8(s+1)}{(s+1)^2 + 4} - \frac{81(2)}{(s+1)^2 + 4} \right]$$

$$\boxed{y(t) = \frac{1}{25} [-8 + 20t + 8\mathbf{e}^{-t} \cos(2t) - 81\mathbf{e}^{-t} \sin(2t)]}$$