Math 3301 **Homework Set 9** **10 Points**

IVP's with Step Functions

Use Laplace transforms to solve the given IVP. In the partial fraction stage all quadratics that can be factored with integer coefficients must be factored.

1. $y'' - 8y' + 16y = 7u_1(t)\mathbf{e}^{4t-4}$ $\quad$ $y(0) = 0, \ y'(0) = 2$

2. $3y'' - y' = u_7(t) - 4u_3(t)\mathbf{e}^{2t-6}$ $\quad$ $y(0) = 0, \ y'(0) = 0$

3. $y'' - y' - 2y = \mathbf{e}^{-3t} + u_4(t)\mathbf{e}^{12-3t}$ $\quad$ $y(0) = 2, \ y'(0) = 0$

Dirac-Delta Function

Use Laplace transforms to solve the given IVP. In the partial fraction stage all quadratics that can be factored with integer coefficients must be factored.

4. $9y'' - 6y' + 10y = 6\delta(t-1)$ $\quad$ $y(0) = -4, \ y'(0) = 1$

5. $y'' + 2y' - 8y = 7\delta(t-3) + 8u_{10}(t)$ $\quad$ $y(0) = 0, \ y'(0) = 0$

Convolution Integrals

6. Find the Laplace Transform of $f(t) = \int_0^t \mathbf{e}^{2t-2\tau}\cos\left(\frac{1}{2}\tau\right)d\tau$.

7. Use a convolution integral (make sure you evaluate the integral!) to find the inverse transform of

$$H(s) = \frac{3}{(s+2)(s-7)}$$

8. Find the solution to the following IVP in terms of $g(t)$.

$$y'' - 8y' + 25y = g(t) \qquad y(0) = -9, \ y'(0) = 0$$