

**2. (3 pts)** Take the Laplace transform of everything.

$$\begin{aligned} 3(s^2Y(s) - sy(0) - y'(0)) - (sY(s) - y(0)) &= \frac{e^{-7s}}{s} - \frac{4e^{-3s}}{s-2} \\ (3s^2 - s)Y(s) &= \frac{e^{-7s}}{s} - \frac{4e^{-3s}}{s-2} \\ Y(s) &= \frac{e^{-7s}}{s^2(3s-1)} - \frac{4e^{-3s}}{s(3s-1)(s-2)} \\ Y(s) &= e^{-7s}F(s) - 4e^{-3s}G(s) \end{aligned}$$

I'll leave it to you to verify the partial fraction work.

$$\begin{aligned} F(s) &= -\frac{3}{s} - \frac{1}{s^2} + \frac{9}{3(s-\frac{1}{3})} & \rightarrow & f(t) = -3 - t + 3e^{\frac{1}{3}t} \\ G(s) &= \frac{1}{10} \left[ \frac{5}{s} - \frac{18}{3(s-\frac{1}{3})} + \frac{1}{s-2} \right] & \rightarrow & g(t) = \frac{1}{10} \left[ 5 - 6e^{\frac{1}{3}t} + e^{2t} \right] \end{aligned}$$

The solution is then,

$$Y(s) = e^{-7s}F(s) - 4e^{-3s}G(s) \quad \boxed{y(t) = u_7(t)f(t-7) - 4u_2(t)g(t-3)}$$

where  $f(t)$  and  $g(t)$  are shown above.

**5. (4 pts)** Take the Laplace transform of everything.

$$\begin{aligned} s^2Y(s) - sy(0) - y'(0) + 2(sY(s) - y(0)) - 8Y(s) &= 7e^{-3s} + \frac{8e^{-10s}}{s} \\ (s^2 + 2s - 8)Y(s) &= 7e^{-3s} + \frac{8e^{-10s}}{s} \\ Y(s) &= \frac{7e^{-3s}}{(s-2)(s+4)} + \frac{8e^{-10s}}{s(s-2)(s+4)} \\ Y(s) &= 7e^{-3s}F(s) + 8e^{-10s}G(s) \end{aligned}$$

I'll leave it to you to verify the partial fraction work.

$$\begin{aligned} F(s) &= \frac{1}{6} \left[ \frac{1}{s-2} - \frac{1}{s+4} \right] & \rightarrow & f(t) = \frac{1}{6} [e^{2t} - e^{-4t}] \\ G(s) &= \frac{1}{24} \left[ -\frac{3}{s} + \frac{2}{s-2} + \frac{1}{s+4} \right] & \rightarrow & g(t) = \frac{1}{24} [-3 + 2e^{2t} + e^{-4t}] \end{aligned}$$

The solution is then,

$$Y(s) = 7e^{-3s}F(s) + 8e^{-10s}G(s) \quad \boxed{y(t) = 7u_3(t)f(t-3) + 8u_{10}g(t-10)}$$

where  $f(t)$  and  $g(t)$  are shown above.

**8. (3 pts)** Take the Laplace transform of everything.

$$s^2Y(s) - sy(0) - y'(0) - 8(sY(s) - y(0)) + 25Y(s) = G(s)$$

$$(s^2 - 8s + 25)Y(s) + 9s - 72 = G(s)$$

$$Y(s) = \frac{G(s)}{s^2 - 8s + 25} + \frac{72 - 9s}{s^2 - 8s + 25}$$

$$Y(s) = G(s)F(s) + H(s)$$

No partial fractions work here, but there is some numerator work involved.

$$F(s) = \frac{\frac{1}{3}}{(s-4)^2 + 9} \rightarrow f(t) = \frac{1}{3}e^{4t} \sin(3t)$$

$$H(s) = \frac{72 - 9(s-4+4)}{(s-4)^2 + 9} = \frac{12(3)}{(s-4)^2 + 9} - \frac{9(s-4)}{(s-4)^2 + 9} \rightarrow h(t) = 12e^{4t} \cos(3t) - 9e^{4t} \sin(3t)$$

The solution is then,

$$Y(s) = G(s)F(s) + H(s)$$

$$\boxed{y(t) = \int \frac{1}{3} g(t-\tau) e^{4\tau} \sin(3\tau) d\tau + 12e^{4t} \cos(3t) - 9e^{4t} \sin(3t)}$$

**Not Graded**

**1.** Take the Laplace transform of everything.

$$s^2Y(s) - sy(0) - y'(0) - 8(sY(s) - y(0)) + 16Y(s) = \frac{7e^{-s}}{s-4}$$

$$(s^2 - 8s + 16)Y(s) - 2 = \frac{7e^{-s}}{s-4}$$

$$(s^2 - 8s + 16)Y(s) = \frac{7e^{-s}}{s-4} + 2$$

$$Y(s) = \frac{7e^{-s}}{(s-4)^3} + \frac{2}{(s-4)^2}$$

$$= 7e^{-s}F(s) + G(s)$$

In both cases here there are no partial fraction work that needs to be done.

$$F(s) = \frac{\frac{1}{2}}{(s-4)^3} \rightarrow f(t) = \frac{1}{2}t^2e^{4t} \quad G(s) = \frac{2}{(s-4)^2} \rightarrow g(t) = 2te^{4t}$$

The solution is then,

$$Y(s) = 7e^{-s}F(s) + G(s) \quad \boxed{y(t) = 7u_1(t)f(t-1) + g(t)}$$

where  $f(t)$  and  $g(t)$  are shown above.

**3.** Take the Laplace transform of everything.

$$\begin{aligned}s^2Y(s) - sy(0) - y'(0) - (sY(s) - y(0)) - 2Y(s) &= \frac{1}{s+3} + \frac{e^{-4s}}{s+3} \\ (s^2 - s - 2)Y(s) - 2s + 2 &= \frac{1 + e^{-4s}}{s+3} \\ Y(s) &= \frac{1 + e^{-4s}}{(s+3)(s-2)(s+1)} + \frac{2s-2}{(s-2)(s+1)} \\ Y(s) &= (1 + e^{-4s})F(s) + G(s)\end{aligned}$$

I'll leave it to you to verify the partial fraction work.

$$\begin{array}{lll} F(s) = \frac{1}{30} \left[ \frac{3}{s+3} + \frac{2}{s-2} - \frac{5}{s+1} \right] & \rightarrow & f(t) = \frac{1}{30} [3e^{-3t} + 2e^{2t} - 5e^{-t}] \\ G(s) = \frac{1}{3} \left[ \frac{2}{s-2} + \frac{4}{s+1} \right] & \rightarrow & g(t) = \frac{1}{3} [2e^{2t} + 4e^{-t}] \end{array}$$

The solution is then,

$$Y(s) = F(s) + e^{-4s}F(s) + G(s) \quad \boxed{y(t) = f(t) + u_4(t)f(t-4) + g(t)}$$

where  $f(t)$  and  $g(t)$  are shown above.

**4.** Take the Laplace transform of everything.

$$\begin{aligned}9(s^2Y(s) - sy(0) - y'(0)) - 6(sY(s) - y(0)) + 10Y(s) &= 6e^{-s} \\ (9s^2 - 6s + 10)Y(s) + 36s - 33 &= 6e^{-s} \\ Y(s) &= \frac{6e^{-s}}{9s^2 - 6s + 10} + \frac{33 - 36s}{9s^2 - 6s + 10} \\ Y(s) &= 6e^{-s}F(s) + G(s)\end{aligned}$$

There is no partial fraction work here, but we do need to do some numerator work.

$$\begin{array}{lll} F(s) = \frac{1}{9(s^2 - \frac{2}{3}s + \frac{10}{9})} = \frac{1}{9} \frac{1}{(s - \frac{1}{3})^2 + 1} & \rightarrow & f(t) = \frac{1}{9} e^{\frac{1}{3}t} \sin(t) \\ G(s) = \frac{1}{9} \frac{33 - 36(s - \frac{1}{3} + \frac{1}{3})}{(s - \frac{1}{3})^2 + 1} = \frac{1}{9} \left[ \frac{21}{(s - \frac{1}{3})^2 + 1} - \frac{36(s - \frac{1}{3})}{(s - \frac{1}{3})^2 + 1} \right] & \rightarrow & g(t) = \frac{1}{9} [21e^{\frac{1}{3}t} \sin(t) - 36e^{\frac{1}{3}t} \cos(t)] \end{array}$$

The solution is then,

$$Y(s) = 6e^{-s}F(s) + G(s) \quad \boxed{y(t) = 6u_1(t)f(t-1) + g(t)}$$

where  $f(t)$  and  $g(t)$  are shown above.

6. This looks like a convolution integral using  $g(t) = e^{2t}$  and  $h(t) = \cos(\frac{1}{2}t)$  so the Laplace transform is then,

$$F(s) = G(s)H(s) = \left( \frac{1}{s-2} \right) \left( \frac{s}{s^2 + \frac{1}{4}} \right) = \boxed{\frac{s}{(s-2)(s^2 + \frac{1}{4})}}$$

7. Rewrite the transform as,

$$H(s) = \frac{3}{s+2} \frac{1}{s-7} = F(s)G(s) \quad \Rightarrow \quad f(t) = 3e^{-2t}, \quad g(t) = e^{7t}$$

Then,

$$h(t) = \int_0^t f(t-\tau)g(\tau)d\tau = \int_0^t 3e^{2\tau-2t}e^{7\tau}d\tau = \int_0^t 3e^{9\tau-2t}d\tau = \frac{1}{3}e^{9\tau-2t} \Big|_0^t = \boxed{\frac{1}{3}(e^{7t} - e^{2t})}$$