Eigenvalues and Eigenvectors

Find the eigenvalues and eigenvectors of the given matrix.

1. $A=\begin{bmatrix} \frac{3}{2} & 5 \\ \frac{1}{2} & 3 \end{bmatrix}$

2. $A=\begin{bmatrix} -2 & 1 \\ -1 & -4 \end{bmatrix}$

3. $A=\begin{bmatrix} 5 & -5 \\ 10 & -9 \end{bmatrix}$

Systems of Differential Equations

Convert the given system into a system of differential equation and give your answer in matrix form.

4. $2y''-9y'-6y=0 \qquad y(0)=3,\ y'(0)=8$

5. $y'''+14y''-y'=0 \qquad y(0)=9,\ y'(0)=-6,\ y''(0)=3$

Real, Distinct Eigenvalues

6. Find the general solution to the following system.

$$\vec{x}'=\begin{bmatrix} 2 & 3 \\ 4 & 9 \end{bmatrix}\vec{x}$$

For problems 7 & 8 solve the system, sketch the phase portrait for the system and determine the stability of the equilibrium solution.

7. $\vec{x}'=\begin{bmatrix} -12 & -15 \\ 2 & 1 \end{bmatrix}\vec{x} \qquad \vec{x}(0)=\begin{pmatrix} 0 \\ -4 \end{pmatrix}$

8. $\vec{x}'=\begin{bmatrix} 4 & 1 \\ 7 & -2 \end{bmatrix}\vec{x} \qquad \vec{x}(0)=\begin{pmatrix} -3 \\ -1 \end{pmatrix}$

Complex Eigenvalues

For problems 9 solve the system, sketch the phase portrait for the system and determine the stability of the equilibrium solution.

9. $\vec{x}'=\begin{bmatrix} -2 & -5 \\ 4 & 2 \end{bmatrix}\vec{x} \qquad \vec{x}(0)=\begin{pmatrix} 7 \\ -4 \end{pmatrix}$

Continued on Back $\Rightarrow$

10. Answer each of the following questions about the given IVP.

(a) Convert the IVP into a system of differential equations and write your answer in matrix form.

(b) Solve the system and use this solution to find the solution to the original 2nd order IVP.

(c) Sketch the phase portrait for the system and determine the stability of the equilibrium solution.

$$4y'' - 4y' + 17y = 0 \qquad y(0) = 5,\ y'(0) = -1$$

Repeated Eigenvalues

For problem 11 & 12 solve the system, sketch the phase portrait for the system and determine the stability of the equilibrium solution.

11. $\vec{x}' = \begin{bmatrix} 1 & -4 \\ 4 & 9 \end{bmatrix}\vec{x} \qquad \vec{x}(0) = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$

12. $\vec{x}' = \begin{bmatrix} -\frac{5}{4} & \frac{3}{2} \\ -\frac{3}{8} & \frac{1}{4} \end{bmatrix}\vec{x} \qquad \vec{x}(0) = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$