Math 2414 **Homework Set 1 – Solutions** **10 Points**

1. (2 pts) $u = 6 - x \quad du = -dx \qquad dv = \mathbf{e}^{4-2x}dx \quad v = -\frac{1}{2}\mathbf{e}^{4-2x}$

$$\int (6-x)\mathbf{e}^{4-2x}\,dx = -\tfrac{1}{2}(6-x)\mathbf{e}^{4-2x} - \tfrac{1}{2}\int \mathbf{e}^{4-2x}\,dx = -\tfrac{1}{2}(6-x)\mathbf{e}^{4-2x} + \tfrac{1}{4}\mathbf{e}^{4-2x} + c$$

Now, let's deal with the limits.

$$\int_2^0 (6-x)\mathbf{e}^{4-2x}\,dx = \left[-\tfrac{1}{2}(6-x)\mathbf{e}^{4-2x} + \tfrac{1}{4}\mathbf{e}^{4-2x}\right]\Big|_2^0 = \boxed{\tfrac{7}{4} - \tfrac{11}{4}\mathbf{e}^4 = -148.3949}$$

5. (2 pts) $u = 3x^4 \quad du = 12x^3dx \quad dv = x^3\cos(2-x^4)\,dx \quad v = -\frac{1}{4}\sin(2-x^4)$

$$\int 3x^7\cos(2-x^4)\,dx = \int 3x^4x^3\cos(2-x^4)\,dx = -\tfrac{3}{4}x^4\sin(2-x^4) + 3\int x^3\sin(2-x^4)\,dx$$
$$= \boxed{-\tfrac{3}{4}x^4\sin(2-x^4) + \tfrac{3}{4}\cos(2-x^4) + c}$$

6. (2 pts)

$$\int \sin^6\left(\tfrac{t}{3}\right)\cos^5\left(\tfrac{t}{3}\right)dt = \int \sin^6\left(\tfrac{t}{3}\right)\left(1-\sin^2\left(\tfrac{t}{3}\right)\right)^2\cos\left(\tfrac{t}{3}\right)dt \qquad u = \sin\left(\tfrac{t}{3}\right)$$
$$= 3\int u^6\left(1-u^2\right)^2du = \boxed{3\left(\tfrac{1}{7}\sin^7\left(\tfrac{t}{3}\right) - \tfrac{2}{9}\sin^9\left(\tfrac{t}{3}\right) + \tfrac{1}{11}\sin^{11}\left(\tfrac{t}{3}\right)\right) + c}$$

8. (2 pts)

$$\int \tan^3(4z)\sec^7(4z)\,dz = \int\left(\sec^2(4z)-1\right)\sec^6(4z)\tan(4z)\sec(4z)\,dz \qquad u = \sec(4z)$$
$$= \tfrac{1}{4}\int\left(u^2-1\right)u^6\,dz = \boxed{\tfrac{1}{4}\left(\tfrac{1}{9}\sec^9(4z) - \tfrac{1}{7}\sec^7(4z)\right) + c}$$

9. (2 pts) Note that for some quotients the ideas that we discussed in this section will also work. Also, we'll need to break the integral up before we really start the process here.

$$\int \frac{1-\sin^3(2t)}{\cos^2(2t)}dt = \int \frac{1}{\cos^2(2t)}dt - \int\frac{\sin^3(2t)}{\cos^2(2t)}dt = \int \sec^2(2t)\,dt - \int\frac{\left(1-\cos^2(2t)\right)\sin(2t)}{\cos^2(2t)}dt$$
$$= \int\sec^2(2t)\,dt + \tfrac{1}{2}\int\frac{1-u^2}{u^2}du = \int\sec^2(2t)\,dt + \tfrac{1}{2}\int u^{-2} - 1\,du$$
$$= \boxed{\tfrac{1}{2}\tan(2t) - \tfrac{1}{2}\left(\sec(2t) + \cos(2t)\right) + c}$$

Not Graded

2. $u = w^2 - 6w \quad du = (2w-6)\,dx \qquad dv = \sin(2w)\,dw \quad v = -\tfrac{1}{2}\cos(2w)$

$$\int (w^2 - 6w)\sin(2w)\,dw = -\tfrac{1}{2}(w^2 - 6w)\cos(2w) + \tfrac{1}{2}\int (2w-6)\cos(2w)\,dw$$

$$u = 2w - 6 \quad du = 2dw \qquad dv = \cos(2w)\,dw \quad v = \tfrac{1}{2}\sin(2w)$$

$$\int (w^2 - 6w)\sin(2w)\,dw = -\tfrac{1}{2}(w^2 - 6w)\cos(2w) + \tfrac{1}{2}\left(\tfrac{1}{2}(2w-6)\sin(2w) - \int \sin(2w)\,dw\right)$$

$$= \boxed{-\tfrac{1}{2}(w^2 - 6w)\cos(2w) + \tfrac{1}{4}(2w-6)\sin(2w) + \tfrac{1}{4}\cos(2w) + c}$$

3. $u = 2\tan^{-1}\left(\tfrac{z}{4}\right) \qquad du = \dfrac{2\left(\frac{1}{4}\right)}{1+\left(\frac{z}{4}\right)^2}dz = \dfrac{\frac{1}{2}}{1+\frac{1}{4}z^2}dz \qquad dv = dz \qquad v = z$

$$\int 2\tan^{-1}\left(\tfrac{z}{4}\right)dz = 2z\tan^{-1}\left(\tfrac{z}{4}\right) - \int \frac{\frac{1}{2}z}{1+\frac{1}{4}z^2}dz = \boxed{2z\tan^{-1}\left(\tfrac{z}{4}\right) - \ln\left(1+\tfrac{1}{4}z^2\right) + c}$$

4. We'll drop the limits to do the integral and then deal with them at the end.

$$u = \cos(4y) \quad du = -4\sin(4y)\,dy \qquad dv = \mathbf{e}^{\frac{y}{2}}dy \qquad v = 2\mathbf{e}^{\frac{y}{2}}$$

$$\int \mathbf{e}^{\frac{y}{2}}\cos(4y)\,dy = 2\mathbf{e}^{\frac{y}{2}}\cos(4y) + 8\int \mathbf{e}^{\frac{y}{2}}\sin(4y)\,dy$$

$$u = \sin(4y) \quad du = 4\cos(4y)\,dy \qquad dv = \mathbf{e}^{\frac{y}{2}}dy \qquad v = 2\mathbf{e}^{\frac{y}{2}}$$

$$\int \mathbf{e}^{\frac{y}{2}}\cos(4y)\,dy = 2\mathbf{e}^{\frac{y}{2}}\cos(4y) + 8\left(2\mathbf{e}^{\frac{y}{2}}\sin(4y) - 8\int \mathbf{e}^{\frac{y}{2}}\cos(4y)\,dy\right)$$

$$= 2\mathbf{e}^{\frac{y}{2}}\cos(4y) + 16\mathbf{e}^{\frac{y}{2}}\sin(4y) - 64\int \mathbf{e}^{\frac{y}{2}}\cos(4y)\,dy$$

$$65\int \mathbf{e}^{\frac{y}{2}}\cos(4y)\,dy = 2\mathbf{e}^{\frac{y}{2}}\cos(4y) + 16\mathbf{e}^{\frac{y}{2}}\sin(4y)$$

$$\int \mathbf{e}^{\frac{y}{2}}\cos(4y)\,dy = \tfrac{1}{65}\left(2\mathbf{e}^{\frac{y}{2}}\cos(4y) + 16\mathbf{e}^{\frac{y}{2}}\sin(4y)\right)$$

Now deal with the limits.

$$\int_0^1 \mathbf{e}^{\frac{y}{2}}\cos(4y)\,dy = \tfrac{1}{65}\left(2\mathbf{e}^{\frac{y}{2}}\cos(4y) + 16\mathbf{e}^{\frac{y}{2}}\sin(4y)\right)\Bigg|_0^1 = \boxed{\tfrac{1}{65}\left[2\mathbf{e}^{\frac{1}{2}}\cos(4) + 16\mathbf{e}^{\frac{1}{2}}\sin(4) - 2\right] = -0.3711}$$

7.

$$\int \cos^4 x\,dx = \int \left(\cos^2 x\right)^2 dx = \int \left(\tfrac{1}{2}\left(1+\cos(2x)\right)\right)^2 dx = \tfrac{1}{4}\int 1+2\cos(2x)+\cos^2(2x)\,dx$$

$$= \tfrac{1}{4}\int 1+2\cos(2x)+\tfrac{1}{2}\left(1+\cos(4x)\right)dx + \tfrac{1}{4}\int \tfrac{3}{2}+2\cos(2x)+\tfrac{1}{2}\cos(4x)\,dx$$

$$= \boxed{\tfrac{1}{4}\left(\tfrac{3}{2}x+\sin(2x)+\tfrac{1}{8}\sin(4x)\right)+c}$$

10. If you can deal with products of tangents and secants you can deal with this problem. The method is nearly identical. Only a few derivatives/signs will change.

$$\int \csc^6(x)\,dx = \int \left(1+\cot^2 x\right)^2 \csc^2(x)\,dx \qquad u=\cot x$$

$$= -\int \left(1+u^2\right)^2 du = \boxed{-\left(\cot x+\tfrac{2}{3}\cot^3 x+\tfrac{1}{5}\cot^5 x\right)+c}$$