**2. (3 pts)**

$$\tfrac{dx}{dy} = 2\left(y+1\right) \quad ds = \sqrt{1+4\left(y+1\right)^2}\,dy \quad x=1: y=-1, x=5: y=1 \quad L=\int_{-1}^{1}\sqrt{1+4\left(y+1\right)^2}\,dy$$

$$y+1=\tfrac{1}{2}\tan\theta \quad dy=\tfrac{1}{2}\sec^2\theta\,d\theta \quad \sqrt{1+4\left(y+1\right)^2}=\sqrt{1+\tan^2\theta}=\left|\sec\theta\right|$$

$$y=-1: \tan\theta=0 \to \theta=0 \qquad y=1: \tan\theta=4 \to \theta=\tan^{-1}\left(4\right)=1.3258$$

$$L=\int_{-1}^{1}\sqrt{1+4\left(y+1\right)^2}\,dy=\tfrac{1}{2}\int_0^{1.3258}\sec^3\theta\,d\theta=\tfrac{1}{4}\left(\sec\theta\tan\theta+\ln\left|\sec\theta+\tan\theta\right|\right)\Big|_0^{1.3258}=4.6462$$

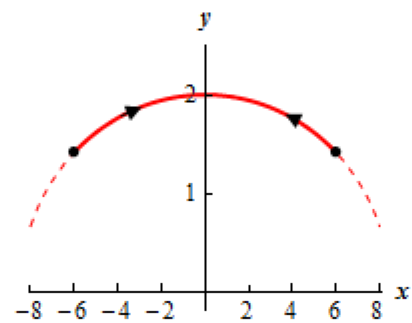
**4. (2 pts)** $\tfrac{dy}{dx}=-4x \quad ds=\sqrt{1+16x^2}$

$$S=\int_0^6 2\pi x\sqrt{1+16x^2}\,dx=\tfrac{\pi}{24}\left(1+16x^2\right)^{\frac{3}{2}}\Big|_0^6=\boxed{\tfrac{\pi}{24}\left(577^{\frac{3}{2}}-1\right)=1814.14}$$

**8. (2 pts)** First eliminate the parameter.

$$\sin\left(3t\right)=\tfrac{x}{6} \qquad \cos^2\left(3t\right)=\tfrac{1}{2}\left(y^2-2\right)$$

$$1=\cos^2\left(\tfrac{t}{4}\right)+\sin^2\left(\tfrac{t}{4}\right)=\tfrac{1}{2}\left(y^2-2\right)+\tfrac{x^2}{36} \quad\to\quad \tfrac{x^2}{72}+\tfrac{y^2}{4}=1$$




So, we will have a portion of this ellipse. Limits on *x* and *y* are,

$$-6\le x\le 6 \qquad \sqrt{2}\le y\le 2$$

We will be at the endpoints at,

$$\text{left}: \sin\left(3t\right)=-1 \to 3t=-\tfrac{\pi}{2}+2\pi k \to t=-\tfrac{\pi}{6}+\tfrac{2}{3}\pi k$$

$$\text{right}: \sin\left(3t\right)=1 \to 3t=\tfrac{\pi}{2}+2\pi k \to t=\tfrac{\pi}{6}+\tfrac{2}{3}\pi k$$

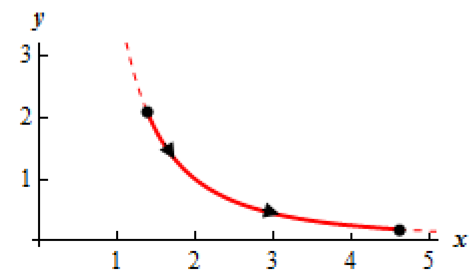
So, the curve will trace out in the range $-\tfrac{\pi}{6}\le t\le\tfrac{\pi}{6}$

We know that sine (and hence x) oscillates and we also know that we won't have the full ellipse (and so we can't get back to the starting point by traveling around the ellipse) and so the only option is for the curve to oscillate as shown above.

**9. (3 pts)** This one is kind of tricky, but remembering logarithm properties and a quick rewrite will make it really easy.

$$x = \ln\left(t^2\right) = 2\ln(t), \qquad y = \frac{1}{\left[\ln(t)\right]^2} = \frac{4}{x^2}$$

Because logarithms are increasing functions we can see that *x* can only increase as *t* increases and *y* can only decrease. So this curve will trace out exactly once in the direction shown in the sketch to the right.




Ranges for *x* and *y* are,

$$\ln(4) \le x \le \ln(100) \qquad \frac{1}{\left[\ln(2)\right]^2} \le y \le \frac{1}{\left[\ln(10)\right]^2}$$

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***Not Graded***

**1.** $\frac{dx}{dy} = 3(2x+3)^{\frac{1}{2}} \qquad ds = \sqrt{1+9(2x+3)}\,dy = \sqrt{18x+28}\,dx$

$$L = \int_0^2 \sqrt{18x+28}\,dx = \tfrac{1}{18}\left(\tfrac{2}{3}\right)(18x+28)^{\frac{3}{2}}\Big|_0^2 = \boxed{\tfrac{1}{27}\left(512 - 28^{\frac{3}{2}}\right) = 13.4755}$$

**3.** This is not as difficult as it looks. This is just an ellipse and so we know that the limits on *x* and *y* are,

$$-\tfrac{1}{3} \le x \le \tfrac{1}{3} \qquad -6 \le y \le 6$$

So, we'll have limits once we get the integral set up. To do that we only need to solve for *x* or *y* and in this case it doesn't really matter which we solve for. I'll do solve for *y* since that will avoid fractions.

$$y = \pm\sqrt{36\left(1-9x^2\right)} = \pm 6\sqrt{1-9x^2}$$

The "+" will give the upper portion of the ellipse and the "-" will give the lower. Also since we can only use a single equation to find the arc length and because ellipses are symmetric we can find the length of the upper and then just double that to get the complete length. This will also mean that we'll use *x* limits of integration. Here's the rest of the work.

$$\frac{dy}{dx} = -27x\left(1-9x^2\right)^{-\frac{1}{2}} = -\frac{27x}{\sqrt{1-9x^2}}$$

$$\sqrt{1+\left(\tfrac{dy}{dx}\right)^2} = \sqrt{1+\tfrac{729x^2}{1-9x^2}} = \sqrt{\tfrac{1+720y^2}{1-9x^2}} \qquad \Rightarrow \qquad \boxed{L = 2\int_{-6}^{6}\sqrt{\tfrac{1+720y^2}{1-9x^2}}\,dx}$$

**5.** Here is the integral for the surface area.

$$\tfrac{dy}{dx} = 3\cos(3x) \qquad A = \int 2\pi y\,ds = 2\pi\int_0^{\pi} \sin(3x)\sqrt{1+9\cos^2(3x)}\,dx$$

This will use the following trig substitution.

$$\cos(3x)=\tfrac{1}{3}\tan\theta \qquad -3\sin(3x)\,dx=\tfrac{1}{3}\sec^2\theta\,d\theta \qquad \sqrt{1+9\cos^2(3x)}=\sqrt{1+\tan^2\theta}=\left|\sec\theta\right|=\sec\theta$$

$$x=0:\ 1=\tfrac{1}{3}\tan\theta \quad\rightarrow\quad \tan\theta=3 \quad\Rightarrow\quad \theta=\tan^{-1}(3)=1.249$$

$$x=\pi:\ -1=\tfrac{1}{3}\tan\theta \quad\rightarrow\quad \tan\theta=-3 \quad\Rightarrow\quad \theta=\tan^{-1}(-3)=-1.249$$

The range of $\theta$'s are in the range from the first to fourth quadrant where cosine is positive and so we can drop the absolute value bars above. The surface area is then,

$$A=\int 2\pi y\,ds=2\pi\int_0^{\pi}\sin(3x)\sqrt{1+9\cos^2(3x)}dx=-\tfrac{2\pi}{9}\int_{1.249}^{-1.249}\sec^3\theta\,d\theta$$

$$=-\tfrac{\pi}{9}\left(\sec\theta\tan\theta+\ln\left|\sec\theta+\tan\theta\right|\right)\Big|_{1.249}^{-1.249}=\boxed{7.89257}$$

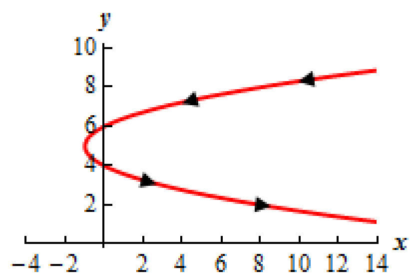
**6.** . $\frac{dx}{dy}=2y\mathbf{e}^{7y}+7y^2\mathbf{e}^{7y} \qquad ds=\sqrt{1+\left(2y\mathbf{e}^{7y}+7y^2\mathbf{e}^{7y}\right)^2}\,dy$

**(a)** $S=\int 2\pi y\,ds=\boxed{\int_0^1 2\pi y\sqrt{1+\left(2y\mathbf{e}^{7y}+7y^2\mathbf{e}^{7y}\right)^2}\,dy}$

**(b)** $S=\int 2\pi x\,ds=\boxed{\int_0^1 2\pi y^2\mathbf{e}^{7y}\sqrt{1+\left(2y\mathbf{e}^{7y}+7y^2\mathbf{e}^{7y}\right)^2}\,dy}$

**7.** In this case to eliminate the parameter we'll need to solve *y* for *t* and plug that into *x*.

$$t=4-y \quad\rightarrow\quad x=(4-y)^2+2(4-y)=y^2-10y+24$$




So, the graph is a parabola that opens to the to the right with vertex $(-1,5)$. A sketch is to the right and we can get the direction by checking the derivative $\frac{dy}{dt}=-1<0$. This is negative so *y* must be decreasing which gives the direction shown in the sketch.

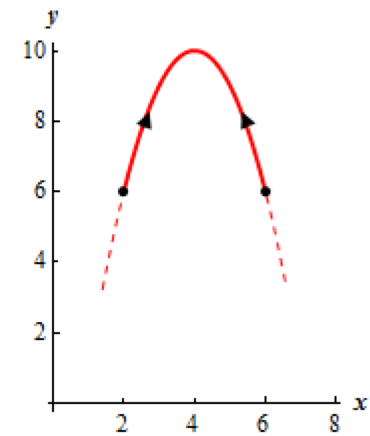
The graph will trace out exactly once since neither *x* or *y* oscillate. From the graph we can see that we have the following limits on *x* and *y*.

$$x\ge -1 \qquad\qquad -\infty<y<\infty$$

**10.** First eliminate the parameter.

$$\cos(6t) = \tfrac{1}{2}(4-x) \qquad \sin^2(6t) = \tfrac{1}{4}(y-6)$$

$$1 = \cos^2\left(\tfrac{t}{4}\right) + \sin^2\left(\tfrac{t}{4}\right) = \tfrac{1}{4}(4-x)^2 + \tfrac{1}{4}(y-6) \quad \rightarrow \quad y = 10 - (4-x)^2$$




So, we will have a portion of this parabola that opens downwards with vertex (4,10). Limits on *x* and *y* are,

$$2 \le x \le 6 \qquad\qquad 6 \le y \le 10$$

We will be at the endpoints at,

$$\text{left}: \ \cos\left(\tfrac{t}{4}\right) = -1 \quad \rightarrow \quad \tfrac{t}{4} = \pi + 2\pi k \quad \rightarrow \quad t = 4\pi + 8\pi k$$

$$\text{right}: \ \cos\left(\tfrac{t}{4}\right) = 1 \quad \rightarrow \quad \tfrac{t}{4} = 0 + 2\pi k \quad \rightarrow \quad t = 8\pi k$$

So, the curve will trace out once in the range $0 \le t \le 4\pi$ and so the curve will trace out

$$\frac{80\pi - (-32\pi)}{4\pi} = 28 \text{ times}$$

We know that cosine (and hence x) oscillates and we also know that we must stay on the parabola and so the only option is for the curve to oscillate as shown above.