3. (2 pts) First the derivative,

$$\frac{dy}{dt} = 3t^2 - 9 \qquad \frac{dx}{dt} = (3-2t)\mathbf{e}^{3t-t^2} \qquad \frac{dy}{dx} = \frac{3t^2-9}{(3-2t)\mathbf{e}^{3t-t^2}}$$

The value of *t* that yields the point is,

$$1 = \mathbf{e}^{3t-t^2} \qquad 3t - t^2 = t(3-t) = 0 \qquad \to \qquad t = 0, 3$$

$$4 = t^3 - 9t + 4 \qquad t^3 - 9t = t(t^2 - 9) = 0 \qquad \to \qquad t = 0, \pm 3$$

So, we'll have two tangent lines at,

$$t = 0: m = \left.\frac{dy}{dt}\right|_{t=0} = -3 \qquad \boxed{y = 4 - 3(x-1) = -3x + 7}$$

$$t = 3: \; m = \left.\frac{dy}{dt}\right|_{t=3} = -6 \qquad \boxed{y = 4 - 6(x-1) = -6x + 10}$$

4. (2 pts) We completely described the path of this particle in **#10** on the previous homework set so we know that this is a portion of as line that traces out once in the range of $0 \le t \le 4\pi$ and will trace out 28 times. Also note that once we have the length of the curve the distance traveled will be 28 times this. The length is then,

$$L = \int_0^{4\pi} \sqrt{\left(\tfrac{1}{2}\sin\left(\tfrac{t}{4}\right)\right)^2 + \left(2\sin\left(\tfrac{t}{4}\right)\cos\left(\tfrac{t}{4}\right)\right)^2}\, dt = \int_0^{4\pi} \sqrt{\tfrac{1}{4}\sin^2\left(\tfrac{t}{4}\right) + 4\sin^2\left(\tfrac{t}{4}\right)\cos^2\left(\tfrac{t}{4}\right)}\, dt$$

$$= \int_0^{4\pi} \tfrac{1}{2}\left|\sin\left(\tfrac{t}{4}\right)\right| \sqrt{1 + 16\cos^2\left(\tfrac{t}{4}\right)}\, dt$$

Now, for $0 \le t \le 4\pi$ we see that $0 \le \frac{t}{4} \le \pi$ and sine is positive here so we can drop the absolute value bars. We'll now need the following trig substitution.

$$\cos\left(\tfrac{t}{4}\right) = \tfrac{1}{4}\tan\theta \qquad -\tfrac{1}{4}\sin\left(\tfrac{t}{4}\right) = \tfrac{1}{4}\sec^2\theta\, d\theta \qquad \sqrt{1 + 16\cos^2\left(\tfrac{t}{4}\right)} = \sqrt{1 + \tan^2\theta} = \left|\sec\theta\right|$$

$$x = 0: \; 1 = \tfrac{1}{4}\tan\theta \qquad \to \qquad \tan\theta = 4 \qquad \Rightarrow \qquad \theta = \tan^{-1}(4) = 1.3258$$

$$t = 4\pi: -1 = \tfrac{1}{4}\tan\theta \qquad \to \qquad \tan\theta = -4 \qquad \Rightarrow \qquad \theta = \tan^{-1}(-4) = -1.3258$$

In this range we are in the 1st and 4th quadrants and so secant is positive. The final answer for the length is then,

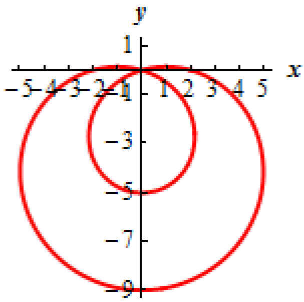
$$L = \int_0^{4\pi} \tfrac{1}{2}\sin\left(\tfrac{t}{4}\right)\sqrt{1 + 16\cos^2\left(\tfrac{t}{4}\right)}\, dt$$

$$= -\tfrac{1}{2}\int_{1.3258}^{-1.3258} \sec^3\theta\, d\theta = -\tfrac{1}{4}\left(\tan\theta\sec\theta + \ln\left|\tan\theta + \sec\theta\right|\right)\Big|_{1.3258}^{-1.3258} = \boxed{9.3064}$$

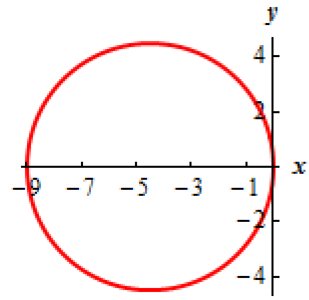
The distance travelled is then : $\boxed{28(9.3064) = 260.5792}$

5. (2 pts) $A = \int 2\pi y\, ds = \boxed{\int_0^3 2\pi\left(t^3 - \mathbf{e}^{7t}\right)\sqrt{\left(\cos(t) - t\sin(t)\right)^2 + \left(3t^2 - 7\mathbf{e}^{7t}\right)^2}\, dt}$

6. (2 pts)



7. (2 pts)



Not Graded

1.

$$\frac{dy}{dt} = 3 + 2\mathbf{e}^{-2t} \qquad \frac{dx}{dt} = 6\mathbf{e}^{6t} \qquad \boxed{\frac{dy}{dx} = \frac{3 + 2\mathbf{e}^{-2t}}{6\mathbf{e}^{6t}} = \tfrac{1}{2}\mathbf{e}^{-6t} + \tfrac{1}{3}\mathbf{e}^{-8t}}$$

$$\frac{d}{dt}\left(\frac{dy}{dx}\right) = -\tfrac{1}{2}\mathbf{e}^{-6t} + \tfrac{1}{3}\mathbf{e}^{-8t} \qquad \boxed{\frac{d^2y}{dx^2} = \frac{-3\mathbf{e}^{-6t} - \frac{8}{3}\mathbf{e}^{-8t}}{6\mathbf{e}^{6t}} = -\tfrac{1}{2}\mathbf{e}^{-12t} - \tfrac{4}{9}\mathbf{e}^{-14t}}$$

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $x = \mathbf{e}^{6t}$, $y = 3t - \mathbf{e}^{-2t}$.

2. First the derivative.

$$\frac{dy}{dt} = 6\sin(3t)\cos(3t) \qquad \frac{dx}{dt} = -2\sin\left(2t - \tfrac{\pi}{3}\right) \qquad \frac{dy}{dx} = \frac{-3\sin(3t)\cos(3t)}{\sin\left(2t - \frac{\pi}{3}\right)}$$

The slope and point at $t = 3$ and tangent line are,

$$m = \left.\frac{dy}{dx}\right|_{t=3} = \frac{-3\sin(\pi)\cos(\pi)}{\sin\left(\frac{\pi}{3}\right)} = 0 \qquad y = 0 \qquad x = \cos\left(\tfrac{\pi}{3}\right) = \tfrac{1}{2} \quad \Rightarrow \quad \boxed{y = 0 + (0)\left(x - \tfrac{1}{2}\right) = 0}$$

8. The limits are,

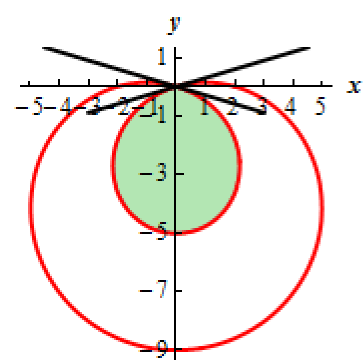
$$0 = 2 - 7\sin\theta \qquad \sin\theta = \tfrac{2}{7} \qquad \theta = \sin^{-1}\left(\tfrac{2}{7}\right) = 0.28975$$

The second angle is $\pi - 0.28975 = 2.85184$. The area is then,

$$A = \tfrac{1}{2}\int_{0.28975}^{2.85184} (2 - 7\sin\theta)^2\, d\theta = \tfrac{1}{2}\int_{0.28975}^{2.85184} 4 - 28\sin\theta + 49\sin^2\theta\, d\theta$$

$$= \tfrac{1}{2}\int_{0.28975}^{2.85184} \tfrac{57}{2} - 28\sin\theta - \tfrac{49}{2}\cos(2\theta)\, d\theta$$

$$= \tfrac{1}{2}\left(\tfrac{57}{2}\theta + 28\cos\theta - \tfrac{49}{4}\sin(2\theta)\right)\Big|_{0.28975}^{2.85184} = \boxed{16.3852}$$



9. The intersection points are,

$$3-2\cos\theta = 5+3\cos\theta \quad \rightarrow \quad \cos\theta = -\tfrac{2}{5}$$

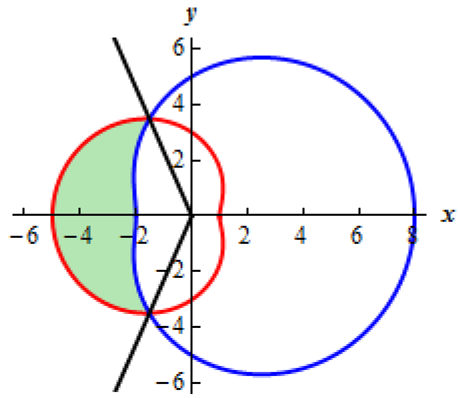
$$\theta = \cos^{-1}\left(-\tfrac{2}{5}\right) = 1.9823$$

The second angle will be $2\pi - 1.9823 = 4.3009$. The area is then,

$$A = \tfrac{1}{2}\int_{1.9823}^{4.3009} \left(3-2\cos\theta\right)^2 - \left(5+3\cos\theta\right)^2 \, d\theta$$

$$= \tfrac{1}{2}\int_{1.9823}^{4.3009} -16 - 42\cos\theta - 5\cos^2\theta \, d\theta$$

$$= \tfrac{1}{2}\int_{1.9823}^{4.3009} -\tfrac{37}{2} - 42\cos\theta - \tfrac{5}{2}\cos\left(2\theta\right) d\theta = \boxed{16.1305}$$



10. This is the area that is inside $r = 3-2\cos\theta$ take away the region from the previous problem. So,

$$A = \tfrac{1}{2}\int_0^{2\pi} \left(3-2\cos\theta\right)^2 d\theta - 16.1305 = \tfrac{1}{2}\int_0^{2\pi} 9 - 12\cos\theta + 4\cos^2\theta \, d\theta - 16.1305$$

$$= \tfrac{1}{2}\int_0^{2\pi} 11 - 12\cos\theta + 2\cos\left(2\theta\right) d\theta - 16.1305 = 11\pi - 16.1305 = \boxed{18.42702}$$