2. (2 pts) To do this a quick rewrite on the sequence terms is needed then we'll see that the sequence **converges** to a value of $\ln\left(\frac{3}{7}\right)$.

$$\lim_{n\to\infty}\left[\ln(3n+9)-\ln(7n+2)\right]=\lim_{n\to\infty}\ln\left(\frac{3n+9}{7n+2}\right)=\ln\left(\frac{3}{7}\right)$$

4. (2 pts) For this sequence we'll need to use the fact from class to see that it **converges** to a value of 0.

$$\lim_{n\to\infty}\left|\frac{(-1)^n n^2}{4n^3+11}\right|=\lim_{n\to\infty}\frac{n^2}{4n^3+11}=0 \quad \text{so by Fact from class we get} \quad \lim_{n\to\infty}\frac{(-1)^n n^2}{4n^3+11}=0$$

6. (2 pts) We'll need the increasing/decreasing information first here and that will require some Calc I.

$$f(x)=\frac{1+2x}{500,000+8x^2} \qquad f'(x)=\frac{-16\left(x^2+x-62,500\right)}{\left(1,000,000+2x^2\right)^2}$$

I'll leave it to you to verify that we get two critical points we get by setting the numerator (and hence the derivative) equal to zero and solving are $x=-250.5$ and $x=249.5$ and that the derivative is positive in the range $0\le x<249.5$ and negative in the range $x>249.5$ and so the function will both increase and decrease for $x\ge 1$ and so the sequence is **not** monotonic.

For bounds we can see that all the sequence terms are positive and so the sequence will be **bounded below** by 0. Also, from our increasing/decreasing work above we know that the function will increase until $x=249.5$ and then decrease. This generally means that either $a_{249}=0.000501$ or $a_{250}=0.000501$ will be the largest sequence term, although in this case they are the same. From this we can see that the sequence will be **bounded above** by 0.000501 (or any larger number of course). The sequence is therefore **bounded**.

11. (2 pts) We need the exponent to be $n-1$ because the series starts at $n=1$.

$$\sum_{n=1}^{\infty}3^{1+2n}\,2^{1-4n}=\sum_{n=1}^{\infty}\frac{3^{2n}3^1}{2^{4n}2^{-1}}=\sum_{n=1}^{\infty}\frac{9^{n-1}9^1 3}{16^{n-1}16^1 2^{-1}}=\sum_{n=1}^{\infty}\frac{27}{8}\left(\frac{9}{16}\right)^{n-1}$$

Okay, because we see $|r|=\frac{9}{16}<1$ and so the series **converges** and its value is,

$$\sum_{n=1}^{\infty}3^{1+2n}\,2^{1-4n}=\frac{\frac{27}{8}}{1-\frac{9}{16}}=\frac{\frac{27}{8}}{\frac{7}{16}}=\boxed{\frac{54}{7}}$$

14. (2 pts) This looks like a telescoping series. I'll leave it to you to verify the partial fractions. The partial sum is,

$$s_n = \sum_{i=2}^{n} \frac{3}{i^2 - 1} = \sum_{i=2}^{n} \left[\frac{\frac{3}{2}}{i-1} - \frac{\frac{3}{2}}{i+1} \right]$$
$$= \left(\frac{\frac{3}{2}}{1} - \frac{\frac{3}{2}}{3} \right) + \left(\frac{\frac{3}{2}}{2} - \frac{\frac{3}{2}}{4} \right) + \left(\frac{\frac{3}{2}}{3} - \frac{\frac{3}{2}}{5} \right) + \left(\frac{\frac{3}{2}}{4} - \frac{\frac{3}{2}}{6} \right) + \left(\frac{\frac{3}{2}}{5} - \frac{\frac{3}{2}}{7} \right) + \cdots$$
$$\cdots + \left(\frac{\frac{3}{2}}{n-5} - \frac{\frac{3}{2}}{n-3} \right) + \left(\frac{\frac{3}{2}}{n-4} - \frac{\frac{3}{2}}{n-2} \right) + \left(\frac{\frac{3}{2}}{n-3} - \frac{\frac{3}{2}}{n-1} \right) + \left(\frac{\frac{3}{2}}{n-2} - \frac{\frac{3}{2}}{n} \right) + \left(\frac{\frac{3}{2}}{n-1} - \frac{\frac{3}{2}}{n+1} \right)$$
$$= \frac{\frac{3}{2}}{1} + \frac{\frac{3}{2}}{2} - \frac{\frac{3}{2}}{n} - \frac{\frac{3}{2}}{n+1} = \frac{9}{4} - \frac{\frac{3}{2}}{n} - \frac{\frac{3}{2}}{n+1}$$

Now,

$$\lim_{n\to\infty} s_n = \lim_{n\to\infty} \left(\frac{9}{4} - \frac{\frac{3}{2}}{n} - \frac{\frac{3}{2}}{n+1} \right) = \frac{9}{4}$$

The limit of the partial sums exists and is finite and so the series will **converge** and its value will be $\frac{9}{4}$.

Not Graded

1. The sequence **converges** to a value of $-\frac{3}{2}$

$$\lim_{n\to\infty} \frac{6n^4 - 9n^2 + 12}{3 - 17n - 8n^3} = -\frac{12}{8} = -\frac{3}{2}$$

3. First write out the first few terms of the series and we'll see that the sequence **diverges**.

$$\left\{ \cos(n\pi) \right\}_{n=0}^{\infty} = \{1, -1, 1, -1, 1, -1, \cdots\} = \left\{ (-1)^n \right\}_{n=0}^{\infty} \qquad \lim_{n\to\infty} (-1)^n \text{ – Does Not Exist}$$

5. Let's first get the increasing/decreasing information. To do this we'll need to do some Calc I.

$$f(x) = \frac{1-x}{3-2x} \qquad f'(x) = \frac{-1}{(3-2x)^2}$$

We can see that the derivative will always be negative for $x \geq 2$ and therefore the function, and hence the sequence, will be decreasing. The sequence is therefore also **monotonic**. Now, because we know that the sequence is decreasing the first term in the sequence will be the largest and so the sequence will be **bounded above** by **1**. Also notice that for $n \geq 2$ the numerator is negative and the denominator is negative and so each sequence term will be positive and so the sequence is **bounded below** by 0. This also means that the sequence is **bounded**.

7. (a) $\displaystyle\sum_{n=0}^{\infty}\frac{1-n}{1+5^n}=\frac{1}{2}+0+\sum_{n=2}^{\infty}\frac{1-n}{1+5^n}$

(b) $\displaystyle 0.439174=\frac{1}{2}+\sum_{n=2}^{\infty}\frac{1-n}{1+5^n} \quad\Rightarrow\quad \boxed{\sum_{n=2}^{\infty}\frac{1-n}{1+5^n}=-0.060826}$

8. $\displaystyle\lim_{n\to\infty}d_n=\lim_{n\to\infty}\frac{6n+8n^2}{9-n^2}=-8$

(a) From the limit above we can see that the sequence **converges** to a value of **-8** .

(b) From the limit above and the **Divergence Test** we can see that the series will **diverge**.

Remember that all we need for a sequence to converge is for the limit of the terms to exist and be a finite value (which we have here). Also, in order for a series to converge the limit of the terms MUST be zero and because we don't have that here the series can't converge.

9. $\displaystyle\lim_{n\to\infty}s_n=\lim_{n\to\infty}\frac{3+\mathbf{e}^{-2n}}{7-6\mathbf{e}^{-2n}}=\frac{3}{7}$. Because the limit of the partial sums exists and is finite the series which generated the partial sums must **converge** and have a value of $\frac{3}{7}$.

10. $\lim_{n\to\infty}12=12\neq 0$ so by the **Divergence Test** the series must **diverge**.

12. The only difference from the first problem is the starting point and so we know that it will **converge** because the value of *r* is the same. To get its value all we need to do is strip out the first three terms and use the value from the first problem.

$$\tfrac{54}{7}=\sum_{n=1}^{\infty}3^{1+2n}\,2^{1-4n}=3^3\,2^{-3}+3^5\,2^{-7}+3^7\,2^{-11}+\sum_{n=4}^{\infty}3^{1+2n}\,2^{1-4n}$$

$$=\tfrac{8379}{2048}+\sum_{n=4}^{\infty}3^{1+2n}\,2^{1-4n} \quad\Rightarrow\quad \boxed{\sum_{n=4}^{\infty}3^{1+2n}\,2^{1-4n}=\tfrac{51939}{14336}}$$

13. This is a harmonic series and so will **diverge**. Remember that it doesn't have to start at $n=1$ to be harmonic. Also we can factor a $-\frac{7}{3}$ out of the series to see that this really is harmonic.