Math 2414

Homework Set 10

3. (2 pts) Because the new line is parallel to the given line the parallel vector is simply : $\vec{v} = \langle -4, 7, 1 \rangle$ and the equation of the line is,

$$\vec{r}(t) = \langle 1, -1, 5 \rangle + t \langle -4, 7, 1 \rangle = \langle 1 - 4t, -1 + 7t, 5 + t \rangle$$

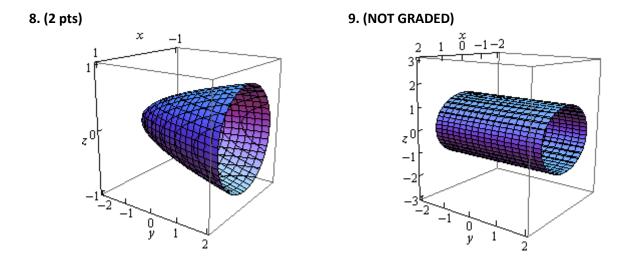
 $x = 1 - 4t$ $y = -1 + 7t$ $z = 5 + t$

4. (3 pts) We know that $\vec{v}_1 = \langle 2, -9, -2 \rangle$ is parallel to the first line and $\vec{v}_2 = \langle 0, 2, 10 \rangle$ is parallel to the second line so if we know the angle between them we will know the angle between the two lines. The angle between them is then,

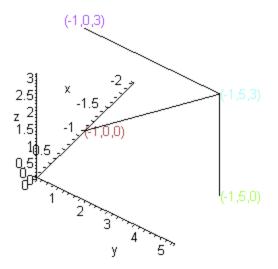
$$\vec{v}_1 \cdot \vec{v}_2 = -38$$
 $\|\vec{v}_1\| = \sqrt{89}$ $\|\vec{v}_2\| = \sqrt{104}$ $\cos \theta = \frac{-38}{\sqrt{89}\sqrt{104}} = -0.3950$ $\theta = 1.9769$ radians

6. (3 pts) Because the plane is orthogonal to the given line then the vector parallel to the line will be orthogonal to the plane. The orthogonal vector is then : $\vec{n} = \langle 5, -1, 3 \rangle$. The equation of the plane is,

$$5(x-6)+(-1)(y+9)+3(z-4)=0 \rightarrow 5x-y+3z=51$$



1. Here's a quick sketch of the given point and the closest point on the plane/axis.



(a) Closest point is (-1, 5, 0) and the distance is $d = \sqrt{(-1+1)^2 + (5-5)^2 + (3-0)^2} = 3$ (b) Closest point is (-1, 0, 3) and the distance is $d = \sqrt{(-1+1)^2 + (5-0)^2 + (3-3)^2} = 5$ (c) Closest point is (-1, 0, 0) and the distance is $d = \sqrt{(-1+1)^2 + (5-0)^2 + (3-0)^2} = \sqrt{34}$

2. For the parallel vector all we need to do is find the vector between the two points or : $\vec{v} = \langle 9, -4, 2 \rangle$. The equation of the line is then (using the second point – although the first point is also valid),

$$\vec{r}(t) = \langle 0, 2, -3 \rangle + t \langle 9, -4, 2 \rangle = \langle 9t, 2 - 4t, -3 + 2t \rangle$$
$$x = 9t \qquad y = 2 - 4t \qquad z = -3 + 2t$$

5. We need a vector normal to this plane and we found that in **#12** on the previous homework. So a normal vector is $\vec{n} = 3\vec{i} + 24\vec{j} + 2\vec{k}$. Using *P* as the point the equation of the plane is then,

$$3(x-9)+24(y-0)+2(z-1)=0 \rightarrow 3x+24y+2z=29$$

7. We can get normal vectors from each plane : $\vec{n}_1 = \langle 6, -1, 4 \rangle$ and $\vec{n}_2 = \langle 6, -1, 0 \rangle$. Now, we can see that these can't be scalar multiplies (check out the third component to see this). Therefore, the normal vectors aren't parallel and this in turn means that the two planes are also not parallel. Next,

$$\vec{n}_1 \cdot \vec{n}_2 = 37 \neq 0$$

The two normal vectors are then not orthogonal and so this means that the two planes are also not orthogonal. Therefore, the two planes are **neither** parallel or orthogonal.