2. (2 pts) Level Curve ($z = c$) : $7c + 6y^2 + 9x^2 - 2 = 0 \Rightarrow 6y^2 + 9x^2 = 2 - 7c$ The level curve is an **ellipse**.

Trace ($x = a$) : $7z + 6y^2 + 9a^2 - 2 = 0 \Rightarrow z = \frac{1}{7}\left(2 - 9a^2 - 6y^2\right)$ A **parabola** in the *yz* –plane.

Trace ($y = b$) : $7z + 6b^2 + 9x^2 - 2 = 0 \Rightarrow z = \frac{1}{7}\left(2 - 6b^2 - 9x^2\right)$ A **parabola** in the *xz* –plane.

3. (2 pts) $\vec{r}(t) = \langle x, y \rangle = \langle h(t), t \rangle = \left\langle 8\mathbf{e}^{1-t^2} - \sqrt{t}, t \right\rangle$

6. (2 pts) Just need to use the formula derived in class and note that the limits on *t* are super important.

$$\vec{r}(t) = (1-t)\langle 2, 0, -7 \rangle + t\langle 1, -1, 4 \rangle \qquad 0 \le t \le 1$$

10. (2 pts) Don't forget the "basics" if integration including integrating by parts and integrating trig functions.

$$\int \vec{r}(t)\,dt = \left\langle \int 9t^2\,dt, \int \sin^2(3t)\,dt, \int t\cos(t)\,dt \right\rangle = \left\langle \int 9t^2 dt, \int \tfrac{1}{2}\left(1 - \cos(6t)\right)dt, t\sin(t) - \int \sin(t)\,dt \right\rangle$$

$$= \boxed{\left\langle 3t^3, \tfrac{1}{2}\left(t - \tfrac{1}{6}\sin(6t)\right), t\sin(t) + \cos(t) \right\rangle + \vec{c}}$$

11. (2 pts) We'll need both the function and its derivative evaluated at $t = -1$.

$$\vec{r}'(t) = 2\vec{i} + \pi\sec^2(\pi t)\,\vec{j} - \mathbf{e}^{8-t}\vec{k}$$

$$r'(-1) = 2\vec{i} + \pi\sec^2(-\pi)\,\vec{j} - \mathbf{e}^9\vec{k} = \underline{2\vec{i} + \pi\vec{j} - \mathbf{e}^9\vec{k}} \qquad\qquad \vec{r}(-1) = \underline{3\vec{i} + \mathbf{e}^9\vec{k}}$$

The tangent line is then,

$$\boxed{\vec{r}(t) = 3\vec{i} + \mathbf{e}^9\vec{k} + t\left(2\vec{i} + \pi\vec{j} - \mathbf{e}^9\vec{k}\right) = \left\langle 3 + 2t, \pi t, \mathbf{e}^9 - \mathbf{e}^9 t \right\rangle}$$

Not Graded

1. Level Curve ($z = c$) : $c = 2y^2 - 5x^2$ So the level curve is a **hyperbola**.

Trace ($x = a$) : $z = 2y^2 - 5a^2$ So this trace is a **parabola** in the *yz* –plane.

Trace ($y = b$) : $z = 2b^2 - 5x^2$ So this trace is a **parabola** in the *xz* –plane.

4. $\vec{r}(t,u) = \langle x, y, z \rangle = \left\langle t, t^4 + \sin(2t - 7u), u \right\rangle$

5. $\vec{r}(t) = \langle 9\cos(t), 8, 9\sin(t) \rangle$

7.(a) $\vec{r}_u \bullet \vec{r}_v = 2\left(3u+v\right)+\left(0\right)\left(v^3\right)+\left(-u^2\right)\left(-3u\right) = \boxed{6u+2v+3u^3}$

$$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3u+v & 0 & -u^2 \\ 2 & v^3 & -3u \end{vmatrix}$$

(b)
$$= 0\vec{i} - 2u^2\,\vec{j} + v^3\left(3u+v\right)\vec{k} - 0\vec{k} - \left(3u+v\right)\left(-3u\right)\vec{j} - \left(-u^2\right)\left(v^3\right)\vec{i}$$
$$= \boxed{u^2v^3\vec{i} + \left(7u^2+3uv\right)\vec{j} + \left(3uv^3+v^4\right)\vec{k}}$$

(c) $\left\|\vec{r}_u \times \vec{r}_v\right\| = \boxed{\sqrt{u^4v^6 + \left(7u^2+3uv\right)^2 + \left(3uv^3+v^4\right)^2}}$

8. Don't forget to L'Hospitals Rule the second term.

$$\lim_{t\to 2}\vec{r}\left(t\right) = \lim_{t\to 2}\left[\mathbf{e}^{1-5t}\,\vec{i} + \frac{1-\cos\left(t-2\right)}{t^2-4}\,\vec{j} - \left(6t^2+t^3\right)\vec{k}\right]$$
$$= \mathbf{e}^{9}\vec{i} + \lim_{t\to 2}\frac{\sin\left(t-2\right)}{2t}\,\vec{j} - \left(12t+2t^2\right)\vec{k} = \boxed{\mathbf{e}^{9}\vec{i} - 32\vec{k}}$$

9. Make sure you can product rule, quotient rule and chain rule!

$$\vec{r}'\left(t\right) = \left\langle \frac{16t}{8t^2+2}, \frac{4-4t^2}{\left(t^2+1\right)^2}, 3\cos\left(6t\right) - 18t\sin\left(6t\right)\right\rangle$$

12. This problem isn't quite as bad as it appears to be. First the derivative (and yes this is messy).

$$\vec{r}'\left(t\right) = \left\langle 0, \tfrac{1}{2}\mathbf{e}^{-4t}\cos\left(\tfrac{t}{2}\right) - 4\mathbf{e}^{-4t}\sin\left(\tfrac{t}{2}\right), -\tfrac{1}{2}\mathbf{e}^{-4t}\sin\left(\tfrac{t}{2}\right) - 4\mathbf{e}^{-4t}\cos\left(\tfrac{t}{2}\right)\right\rangle$$

Now the magnitude and with proper simplification this is really simple.

$$\left\|\vec{r}'\left(t\right)\right\| = \sqrt{0^2 + \left(\tfrac{1}{2}\mathbf{e}^{-4t}\cos\left(\tfrac{t}{2}\right) - 4\mathbf{e}^{-4t}\sin\left(\tfrac{t}{2}\right)\right)^2 + \left(-\tfrac{1}{2}\mathbf{e}^{-4t}\sin\left(\tfrac{t}{2}\right) - 4\mathbf{e}^{-4t}\cos\left(\tfrac{t}{2}\right)\right)^2}$$
$$= \sqrt{\mathbf{e}^{-8t}\left(\tfrac{65}{4}\cos^2\left(3t\right) + \tfrac{65}{4}\sin^2\left(3t\right)\right)} = \underline{\tfrac{1}{2}\sqrt{65}\,\mathbf{e}^{-4t}}$$

Now, the unit tangent vector is,

$$\vec{T}\left(t\right) = \frac{2}{\sqrt{65}\,\mathbf{e}^{-4t}}\left\langle 0, \tfrac{1}{2}\mathbf{e}^{-4t}\cos\left(\tfrac{t}{2}\right) - 4\mathbf{e}^{-4t}\sin\left(\tfrac{t}{2}\right), -\tfrac{1}{2}\mathbf{e}^{-4t}\sin\left(\tfrac{t}{2}\right) - 4\mathbf{e}^{-4t}\cos\left(\tfrac{t}{2}\right)\right\rangle$$
$$= \boxed{\tfrac{1}{\sqrt{65}}\left\langle 0, \cos\left(\tfrac{t}{2}\right) - 8\sin\left(\tfrac{t}{2}\right), -\sin\left(\tfrac{t}{2}\right) - 8\cos\left(\tfrac{t}{2}\right)\right\rangle}$$

Now we need the derivative this and its magnitude .

$$\vec{T}'\left(t\right) = \tfrac{1}{\sqrt{65}}\left\langle 0, -\tfrac{1}{2}\sin\left(\tfrac{t}{2}\right) - 4\cos\left(\tfrac{t}{2}\right), -\tfrac{1}{2}\cos\left(\tfrac{t}{2}\right) + 4\sin\left(\tfrac{t}{2}\right)\right\rangle \qquad \left\|\vec{T}'\left(t\right)\right\| = \tfrac{1}{2}$$

The unit normal vector is then,

$$\boxed{\vec{N}(t) = \tfrac{2}{\sqrt{65}}\left\langle 0, -\tfrac{1}{2}\sin\left(\tfrac{t}{2}\right) - 4\cos\left(\tfrac{t}{2}\right), -\tfrac{1}{2}\cos\left(\tfrac{t}{2}\right) + 4\sin\left(\tfrac{t}{2}\right)\right\rangle}$$