4. (2 pts) This function is not continuous at (0,0) so let's see if we can find a couple of paths that give different values for the limit.

***y*-axis ($x = 0$) :** $\displaystyle \lim_{(x,y)\to(0,0)} \frac{3xy^4}{2x^2+5y^8} = \lim_{(0,y)\to(0,0)} \frac{0}{5y^8} = \underline{0}$

$x = y^4$ **:** $\displaystyle \lim_{(x,y)\to(0,0)} \frac{3xy^4}{2x^2+5y^8} = \lim_{(y^4,y)\to(0,0)} \frac{3y^4y^4}{2y^8+5y^8} = \lim_{(x,x^4)\to(0,0)} \frac{3}{7} = \underline{\frac{3}{7}}$

So, we have two paths that give different values of the limit and so we know that $\displaystyle \lim_{(x,y)\to(0,0)} \frac{3xy^4}{2x^2+5y^8}$ doesn't exist.

5. (2 pts)

$$g_x = 4x^3y^2z - 6x\cos^2\left(x^2\right)\sin\left(x^2\right) \qquad g_y = 2x^4yz - 2y\cos\left(8z - y^2\right)$$

$$g_z = x^4y^2 + 8\cos\left(8z - y^2\right)$$

6. (2 pts) $w_x = -\mathbf{e}^{8y^2-x^2} - 2x\left(z^2 - x\right)\mathbf{e}^{8y^2-x^2} \qquad w_y = 16y\left(z^2 - x\right)\mathbf{e}^{8y^2-x^2} \qquad w_z = 2z\mathbf{e}^{8y^2-x^2}$

10. (2 pts)

$$h_u = 2uv^2\mathbf{e}^{u^2v^2} + \frac{8}{v} \qquad h_v = 2u^2v\mathbf{e}^{u^2v^2} - \frac{8u}{v^2}$$

$$h_{uu} = 2v^2\mathbf{e}^{u^2v^2} + 4u^2v^4\mathbf{e}^{u^2v^2} \qquad h_{uv} = 4uv\mathbf{e}^{u^2v^2} + 4u^3v^3\mathbf{e}^{u^2v^2} - \frac{8}{v^2}$$

$$h_{vu} = 4uv\mathbf{e}^{u^2v^2} + 4u^3v^3\mathbf{e}^{u^2v^2} - \frac{8}{v^2} \qquad h_{vv} = 2u^2\mathbf{e}^{u^2v^2} + 4u^4v^2\mathbf{e}^{u^2v^2} + \frac{16u}{v^3}$$

11. (2 pts) Because we can do the integrals in any order we'll do them in a different order than that requested to make our life a little easier.

$$f_u = -3u^2t^{20} + \cos\left(9t^5 + 8t^3 - 14t^2 + t - 1\right)$$

$$f_{uu} = -6ut^{20}$$

$$f_{uut} = -120ut^{19}$$

$$f_{uutt} = -2280ut^{18}$$

$$\boxed{f_{uuttt} = -41040ut^{17} = f_{ttuut}}$$

Not Graded

1. This is the function we used in **12** from the first homework set so all we need use the work from the last homework set and do the integral.

$$L = \int_1^6 \left\| r'(t) \right\| dt = \int_1^6 \tfrac{1}{2}\sqrt{65}\,\mathbf{e}^{-4t}\,dt = \left(-\tfrac{1}{8}\sqrt{65}\,\mathbf{e}^{-4t}\right)\Big|_1^6 = \boxed{\tfrac{\sqrt{65}}{8}\left(\mathbf{e}^{-4} - \mathbf{e}^{-24}\right)}$$

2. Not much to do here. This function is continuous at (0,0) so : $\displaystyle\lim_{(x,y)\to(0,0)} \frac{5 - x + 8y}{x^2 + y^2 - 8xy + 2} = \boxed{\frac{5}{2}}$

3. This function is not continuous at (0,0) so let's see if we can find a couple of paths that give different values for the limit.

***x*-axis ($y = 0$) :** $\displaystyle\lim_{(x,y)\to(0,0)} \frac{6y^4 + x^4}{(2x - 7y)^4} = \lim_{(x,0)\to(0,0)} \frac{x^4}{(2x)^4} = \lim_{(x,0)\to(0,0)} \frac{1}{16} = \underline{\frac{1}{16}}$

***y*-axis ($x = 0$) :** $\displaystyle\lim_{(x,y)\to(0,0)} \frac{6y^4 + x^4}{(2x - 7y)^4} = \lim_{(0,y)\to(0,0)} \frac{6y^4}{(-7y)^4} = \lim_{(0,y)\to(0,0)} \frac{6}{2401} = \underline{\frac{6}{2401}}$

So, we have two paths that give different values of the limit and so we know that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{6y^4 + x^4}{(2x - 7y)^4}$ doesn't exist.

7.

$$f_u = 4u^3 \ln\left(\frac{s}{3v}\right) + 2uv^3 \sec\left(u^2v^3\right)\tan\left(u^2v^3\right) \qquad f_v = -\frac{u^4}{v} + 3u^2v^2 \sec\left(u^2v^3\right)\tan\left(u^2v^3\right)$$

$$f_s = \frac{u^4}{s} \qquad f_t = 0$$

8. First let's find $\dfrac{\partial z}{\partial x}$

$$2z\mathbf{e}^{z^2} \frac{\partial z}{\partial x} = \frac{\partial z}{\partial x}\tan(4 - x) - z\sec^2(4 - x)$$

$$\left(\tan(4 - x) - 2z\mathbf{e}^{z^2}\right)\frac{\partial z}{\partial x} = z\sec^2(4 - x) \qquad \Rightarrow \qquad \boxed{\frac{\partial z}{\partial x} = \frac{z\sec^2(4 - x)}{\tan(4 - x) - 2z\mathbf{e}^{z^2}}}$$

Now $\dfrac{\partial z}{\partial y}$

$$3y^2 + 2z\mathbf{e}^{z^2}\frac{\partial z}{\partial y} = \frac{\partial z}{\partial y}\tan\left(4-x\right)$$

$$3y^2 = \left(\tan\left(4-x\right) - 2z\mathbf{e}^{z^2}\right)\frac{\partial z}{\partial y} \quad \Rightarrow \quad \boxed{\frac{\partial z}{\partial y} = \frac{3y^2}{\tan\left(4-x\right) - 2z\mathbf{e}^{z^2}}}$$

9.

$$z_x = 2x^{-3} - \frac{1}{x} \qquad z_y = \tfrac{3}{2}y^{\frac{1}{2}} - \frac{1}{y}$$

$$z_{xx} = -6x^{-4} + \frac{1}{x^2} \qquad z_{xy} = 0 \qquad z_{yx} = 0 \qquad z_{yy} = \tfrac{3}{4}y^{-\frac{1}{2}} + \frac{1}{y^2}$$

12. Order probably won't matter much here so we'll do the derivatives in the requested order.

$$\frac{\partial u}{\partial z} = 16x^4y^{-3}z - \frac{1}{z} \qquad \frac{\partial^2 u}{\partial y \partial z} = -48x^4y^{-4}z \qquad \frac{\partial^3 u}{\partial z \partial y \partial z} = -48x^4y^{-4}$$

$$\frac{\partial^4 u}{\partial x \partial z \partial y \partial z} = -192x^3y^{-4} \qquad \boxed{\frac{\partial^5 u}{\partial x^2 \partial z \partial y \partial z} = -576x^2y^{-4}}$$