Math 2415 **Homework Set 3** **10 Points**

Differentials

1. Find the differential for the function $u = \mathbf{e}^{\frac{2x}{y}} + \sqrt{z}\sin(xy)$

Chain Rule

2. Use the Chain Rule to find $\dfrac{dz}{dq}$ given that,

$$z = 2v + x^2w^3 - y^2 \qquad x = 2q^4 \quad y = \ln(7-q) \quad v = \tan(2q) \quad w = 4q$$

3. Use the Chain Rule to find $\dfrac{\partial w}{\partial s}$ and $\dfrac{\partial w}{\partial t}$ given that,

$$w = x^2\cos(2y) - z^3 + y^2, \qquad x = t^5 \quad y = s\mathbf{e}^{6t} \quad z = p^2 \qquad p = s^2t^3$$

4. Use the Chain Rule to find formulas for $\dfrac{\partial w}{\partial p}$ and $\dfrac{\partial w}{\partial q}$ given that,

$$w = w(x,y,z) \quad x = x(s,t) \quad y = y(s,p) \quad z = z(q) \quad s = s(p) \quad t = t(p,q)$$

5. Use the Chain Rule to find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ for $y^3 + \mathbf{e}^{z^2} = z\tan(4-x)$

Directional Derivatives

6. Find ∇f and the directional derivative for $f(x,y) = y^2\sin(x^2y)$ in the direction of $\vec{v} = \langle 5,-3\rangle$ at the point $(-2,4)$.

7. Find the directional derivative of $f(x,y,z) = 4y\mathbf{e}^{-x^2} + \dfrac{8z}{y^2}$ in the direction of $\vec{v} = \langle 3,2,-1\rangle$.

8. Find the maximum rate of change of $f(x,y,z) = \dfrac{2x}{y} - \dfrac{y}{2z}$ at the point $\left(-7,-1,\tfrac{1}{2}\right)$ and the direction in which it occurs.

9. Given that $\vec{u} = \left\langle \dfrac{3}{5}, -\dfrac{4}{5}\right\rangle$, $\vec{v} = \left\langle -\dfrac{2}{\sqrt{13}}, -\dfrac{3}{\sqrt{13}}\right\rangle$, $\vec{w} = \left\langle \dfrac{1}{\sqrt{17}}, \dfrac{4}{\sqrt{17}}\right\rangle$, $D_{\vec{u}}f(7,8) = \dfrac{27}{10}$ and $D_{\vec{v}}f(7,8) = \dfrac{8}{\sqrt{13}}$ determine the value of $D_{\vec{w}}f(2,1)$.