2. (2 pts)

$$\begin{aligned}\frac{dz}{dq} &= \frac{dz}{dx}\frac{dx}{dq} + \frac{dz}{dy}\frac{dy}{dq} + \frac{dz}{dv}\frac{dv}{dq} + \frac{dz}{dw}\frac{dw}{dq} \\ &= \left(2xw^3\right)\left(8q^3\right) + \left(-2y\right)\left(\tfrac{-1}{7-q}\right) + 2\left(2\sec^2\left(2q\right)\right) + \left(3x^2w^2\right)\left(4\right) \\ &= \boxed{16xw^3q^3 + \tfrac{2y}{7-q} + 4\sec^2\left(2q\right) + 12x^2w^2} \\ &= \boxed{2816q^{10} + \tfrac{2\ln(7-q)}{7-q} + 4\sec^2\left(2q\right)}\end{aligned}$$

Note that you were not required to plug back in for *x*, *y* or *z*.

4. (3 pts)

$$\boxed{\begin{aligned}\frac{\partial w}{\partial p} &= \frac{\partial w}{\partial x}\frac{\partial x}{\partial s}\frac{ds}{dp} + \frac{\partial w}{\partial x}\frac{\partial x}{\partial t}\frac{\partial t}{\partial p} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s}\frac{ds}{dp} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial p} \\ \frac{\partial w}{\partial q} &= \frac{\partial w}{\partial x}\frac{\partial x}{\partial t}\frac{\partial t}{\partial q} + \frac{\partial w}{\partial z}\frac{dz}{dq}\end{aligned}}$$

6. (2 pts) Don't forget that the vector needs to be a unit vector.

$$\nabla f\left(x,y\right) = \left\langle 2xy^3\cos\left(x^2y\right), 2y\sin\left(x^2y\right) + x^2y^2\cos\left(x^2y\right)\right\rangle$$

$$\nabla f\left(-2,4\right) = \left\langle -256\cos\left(16\right), 8\sin\left(16\right) + 64\cos\left(16\right)\right\rangle \qquad \vec{u} = \left\langle \frac{5}{\sqrt{34}}, -\frac{3}{\sqrt{34}}\right\rangle$$

$$\begin{aligned}D_{\vec{u}} f\left(-2,4\right) &= \left\langle -256\cos\left(16\right), 8\sin\left(16\right) + 64\cos\left(16\right)\right\rangle \bullet \left\langle \frac{5}{\sqrt{34}}, -\frac{3}{\sqrt{34}}\right\rangle \\ &= \boxed{-\frac{1}{\sqrt{34}}\left(1472\cos\left(16\right) + 24\sin\left(16\right)\right) = 242.942}\end{aligned}$$

9. (3 pts) Notice that all the vectors are unit vectors. Now all we need to do is set up the equations,

$$D_{\vec{u}} f\left(7,8\right) = \frac{3}{5} f_x\left(7,8\right) - \frac{4}{5} f_y\left(7,8\right) = \frac{27}{10}$$

$$D_{\vec{v}} f\left(7,8\right) = -\frac{2}{\sqrt{13}} f_x\left(7,8\right) - \frac{3}{\sqrt{13}} f_y\left(7,8\right) = \frac{8}{\sqrt{13}}$$

This is a system of equations that we can solve for the "unknowns" $f_x\left(7,8\right)$ and $f_y\left(7,8\right)$. Doing so gives $f_x\left(7,8\right) = \frac{1}{2}$ and $f_y\left(7,8\right) = -3$. We can now compute,

$$D_{\vec{w}} f\left(7,8\right) = \frac{1}{\sqrt{17}}\left(\frac{1}{2}\right) + \frac{4}{\sqrt{17}}\left(-3\right) = \boxed{\frac{-23}{2\sqrt{17}}}$$

Not Graded

1. $$\boxed{du = \left(\tfrac{2}{y}\mathbf{e}^{\frac{2x}{y}} + y\sqrt{z}\cos(xy)\right)dx + \left(-\tfrac{2x}{y^2}\mathbf{e}^{\frac{2x}{y}} + x\sqrt{z}\cos(xy)\right)dy + \tfrac{1}{2}z^{-\frac{1}{2}}\sin(xy)\,dz}$$

3.

$$\begin{aligned}\frac{\partial w}{\partial s} &= \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial w}{\partial z}\frac{dz}{dp}\frac{\partial p}{\partial s}\\ &= 2x\cos(2y)(0) + \left(-2x^2\sin(2y) + 2y\right)\left(\mathbf{e}^{6t}\right) - 3z^2(2p)\left(2st^3\right)\\ &= \boxed{\mathbf{e}^{6t}\left(-2t^{10}\sin\left(2s\mathbf{e}^{6t}\right) + 2s\mathbf{e}^{6t}\right) - 12s^{11}t^{18}}\end{aligned}$$

$$\begin{aligned}\frac{\partial w}{\partial t} &= \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}\frac{dz}{dp}\frac{\partial p}{\partial t}\\ &= 2x\cos(2y)\left(5t^4\right) + \left(-2x^2\sin(2y) + 2y\right)\left(6s\mathbf{e}^{6t}\right) - 3z^3(2p)\left(3s^2t^2\right)\\ &= \boxed{10t^9\cos\left(2s\mathbf{e}^{6t}\right) + 6s\mathbf{e}^{6t}\left(-2t^{10}\sin\left(2s\mathbf{e}^{6t}\right) + 2s\mathbf{e}^{6t}\right) - 18s^{12}t^{17}}\end{aligned}$$

5.

$$y^3 + \mathbf{e}^{z^2} - z\tan(4-x) = 0$$

$$\boxed{\frac{\partial z}{\partial x} = -\frac{z\sec^2(4-x)}{2z\mathbf{e}^{z^2} - \tan(4-x)} = \frac{z\sec^2(4-x)}{\tan(4-x) - 2z\mathbf{e}^{z^2}}}$$

$$\boxed{\frac{\partial z}{\partial y} = -\frac{3y^2}{2z\mathbf{e}^{z^2} - \tan(4-x)} = \frac{3y^2}{\tan(4-x) - 2z\mathbf{e}^{z^2}}}$$

7. Don't forget that the vector needs to be a unit vector.

$$\nabla f = \left\langle -8xy\mathbf{e}^{-x^2}, 4\mathbf{e}^{-x^2} - \frac{16z}{y^3}, \frac{8}{y^2}\right\rangle \qquad \vec{u} = \left\langle \frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{-1}{\sqrt{14}}\right\rangle$$

$$\boxed{D_{\vec{u}}f(x,y) = \frac{1}{\sqrt{14}}\left(-24xy\mathbf{e}^{-x^2} + 8\mathbf{e}^{-x^2} - \frac{32z}{y^3} - \frac{8}{y^2}\right)}$$

8. We'll need the gradient for this.

$$\nabla f(x,y,z) = \left\langle \frac{2}{y}, -\frac{2x}{y^2} - \frac{1}{2z}, \frac{y}{2z^2}\right\rangle \qquad \nabla f\left(-7,-1,\tfrac{1}{2}\right) = \langle -2, 13, -2\rangle \qquad \left\|\nabla f\left(-7,-1,\tfrac{1}{2}\right)\right\| = \sqrt{177}$$

So, the maximum rate of change of the function is $\sqrt{177}$ and it occurs in the direction of $\langle -2,13,-2\rangle$.