1. (2 pts)

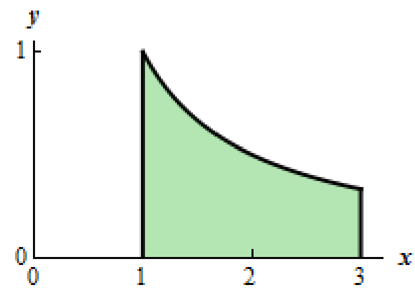
$$\int_1^{-2}\int_0^1 x^3 y^4 \mathbf{e}^{x^2y^5}\,dy\,dx = \int_1^{-2} \tfrac{1}{5} x\mathbf{e}^{x^2y^5}\Big|_0^1\,dx \qquad u = x^2y^5$$

$$= \tfrac{1}{2}\int_1^{-2} \tfrac{1}{5}\left(x\mathbf{e}^{x^2} - x\right)dx$$

$$= \tfrac{1}{5}\left(\tfrac{1}{2}\mathbf{e}^{x^2} - \tfrac{1}{2}x^2\right)\Big|_1^{-2} = \boxed{\tfrac{1}{10}\left(\mathbf{e}^4 - \mathbf{e} - 3\right)}$$

6. (2 pts) A quick sketch of the region is to the right and by looking at this and the integral is looks like we need to integrate with respect to *y* first and so the limits for the integral will be,

$$0 \le y \le \tfrac{1}{x} \qquad 1 \le x \le 3$$



The integral is then,

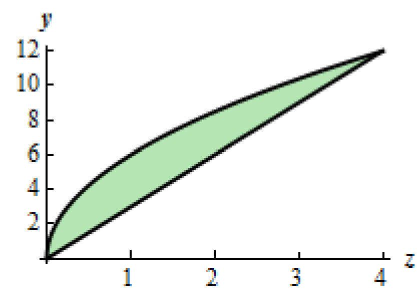
$$\iint\limits_D \sin\left(1+\ln(x)\right)dA = \int_1^3\int_0^{\frac{1}{x}} \sin\left(1+\ln(x)\right)dy\,dx$$

$$= \int_1^3 \tfrac{1}{x}\sin\left(1+\ln(x)\right)dx$$

$$= \boxed{\cos(1) - \cos\left(1+\ln(3)\right) = 1.0440}$$

8. (2 pts) The basic formula here is the same except now the region *D* is in the *yz*-plane. A sketch of the region is to the right and the order of integration really doesn't seem to matter here so we use the following order/limits.

$$3z \le y \le 6\sqrt{z} \qquad 0 \le z \le 4$$



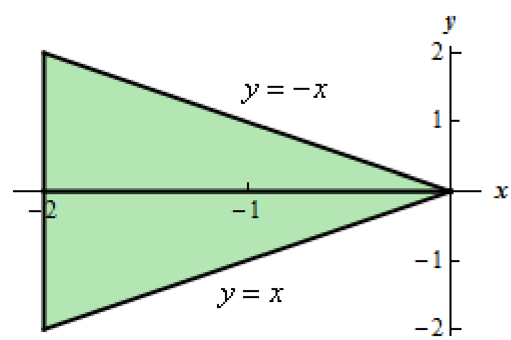
Here's the volume.

$$V = \iint\limits_D 1 + y^2 + 4z^2\,dA = \int_0^4\int_{3z}^{6\sqrt{z}} 1 + y^2 + 4z^2\,dy\,dz$$

$$= \int_0^4 6z^{\frac{1}{2}} + 72z^{\frac{3}{2}} + 24z^{\frac{5}{2}} - 3z - 21z^3\,dz = \boxed{\tfrac{16216}{35} = 463.314}$$

9. (2 pts) The region is sketched to the right. The original limits are : $x \le y \le -x \quad -2 \le x \le 0$. Interchanging limits will require us to do two integrals. The limits are,

$$-2 \le x \le -y, \quad 0 \le y \le 2$$
$$-2 \le x \le y, \; -2 \le y \le 0$$



Here is the integral,

$$\int_{-2}^{0}\int_{x}^{-x} 12x^2y^4\,dy\,dx = \int_{0}^{2}\int_{-2}^{-y} 12x^2y^4\,dx\,dy + \int_{-2}^{0}\int_{-2}^{y} 12x^2y^4\,dx\,dy$$
$$= \int_{0}^{2} 32y^4 - 4y^7\,dy + \int_{-2}^{0} 4y^7 + 32y^4\,dy = \tfrac{384}{5} + \tfrac{384}{5} = \boxed{\tfrac{768}{5}}$$

11. (2 pts) Note that the equation of the hypotenuse for each triangle is $y = -x$ and we will be doing two integrals for this problem. Each region seems to be set up to do *x* first and then *y* so here are the limits.

$$0 \le y \le -x \qquad\qquad -3 \le y \le -x$$
$$-3 \le x \le 0 \qquad\qquad 0 \le x \le 3$$

Here's the integral

$$\iint_D x(1-4y)\,dA = \int_{-3}^{0}\int_{0}^{-x} x(1-4y)\,dy\,dx + \int_{0}^{3}\int_{-3}^{-x} x(1-4y)\,dy\,dx$$
$$= \int_{-3}^{0} -2x^3 - x^2\,dx + \int_{0}^{3} 21x - x^2 - 2x^3\,dx = \tfrac{63}{2} + 45 = \boxed{\tfrac{153}{2}}$$

Not Graded

2. It won't matter which we integrate with respect to first here so we'll do *y* first. Potentially less mess.

$$\iint_R 6y\cos^2(4x) + \frac{12y^3x^2}{y^4+2}\,dA = \int_{-1}^{0}\int_{0}^{3} 6y\cos^2(4x) + \frac{12y^3x^2}{y^4+2}\,dy\,dx$$
$$= \int_{-1}^{0} \left(3y^2\cos^2(4x) + 3x^2\ln\left(y^4+2\right)\right)\Big|_0^3\,dx$$
$$= \int_{-1}^{0} 27\cos^2(4x) + 3x^2\left(\ln(83) - \ln(2)\right)dx$$
$$= \int_{-1}^{0} \tfrac{27}{2}\left(1+\cos(8x)\right) + 3\ln\left(\tfrac{83}{2}\right)x^2\,dx$$
$$= \left(\tfrac{27}{2}\left(x + \tfrac{1}{8}\sin(8x)\right) + \ln\left(\tfrac{83}{2}\right)x^3\right)\Big|_{-1}^{0}$$
$$= \boxed{\tfrac{27}{2}\left(1 - \tfrac{1}{8}\sin(-8)\right) + \ln\left(\tfrac{83}{2}\right) = 18.8952}$$

3. Here integrating *x* first will almost certainly reduce the amount of work because we'll only need to integrate by parts once as opposed to twice if we integrate *y* first.

$$\iint_R x\sin(4y-x)\,dA = \int_0^3\int_{-2}^0 x\sin(4y-x)\,dx\,dy$$
$$= \int_0^3 \left(x\cos(4y-x)+\sin(4y-x)\right)\Big|_{-2}^0 dy$$
$$= \int_0^3 \sin(4y)+2\cos(4y+2)-\sin(4y+2)\,dy$$
$$= \left(-\tfrac{1}{4}\cos(4y)+\tfrac{1}{2}\sin(4y+2)+\tfrac{1}{4}\cos(4y+2)\,dy\right)\Big|_0^3$$
$$= \boxed{\tfrac{1}{4}\left[\cos(14)+2\sin(14)-\cos(12)-\cos(2)-2\sin(2)+1\right]=0.2179}$$

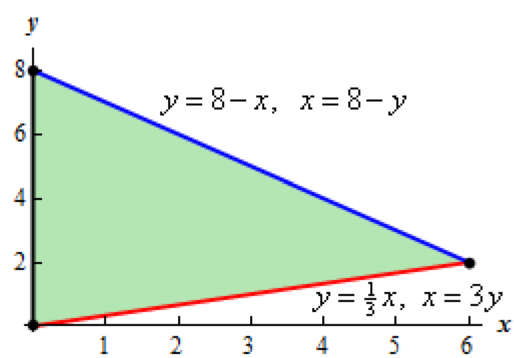
4.

$$\int_0^2\int_{2x+1}^{x^3} 3+20y^3\,dy\,dx = \int_0^2 \left(3y+5y^4\right)\Big|_{2x+1}^{x^3} dx$$
$$= \int_0^2 3x^3+5x^{12}-\left[3(2x+1)+5(2x+1)^4\right]dx$$
$$= \left(\tfrac{3}{4}x^4+\tfrac{5}{13}x^{13}-\tfrac{3}{4}(2x+1)^2-\tfrac{1}{2}(2x+1)^5\right)\Big|_0^2$$
$$= \tfrac{82239}{52}-\left(-\tfrac{5}{4}\right)=\boxed{\tfrac{20576}{13}=1582.77}$$

5.

$$\iint_D y^4\mathbf{e}^{2+x^4}\,dA = \int_1^2\int_0^{x^{\frac{3}{5}}} y^4\mathbf{e}^{2+x^4}\,dy\,dx = \int_1^2 \tfrac{1}{5}y^5\mathbf{e}^{2+x^4}\Big|_0^{x^{\frac{3}{5}}} dx = \int_1^2 \tfrac{1}{5}x^3\mathbf{e}^{2+x^4}\,dx = \tfrac{1}{20}\mathbf{e}^{2+x^4}\Big|_1^2 = \boxed{\tfrac{1}{20}\left(\mathbf{e}^{18}-\mathbf{e}^3\right)}$$

7. A sketch of the region is to the right. Here are the limits we'll need for each part.



(a) $\frac{1}{3}x \le y \le 8-x \qquad 0 \le x \le 6$

(b) We'll need two regions for this one.

$0 \le x \le 3y \qquad 0 \le y \le 2$

$0 \le x \le 8-y \qquad 2 \le y \le 8$

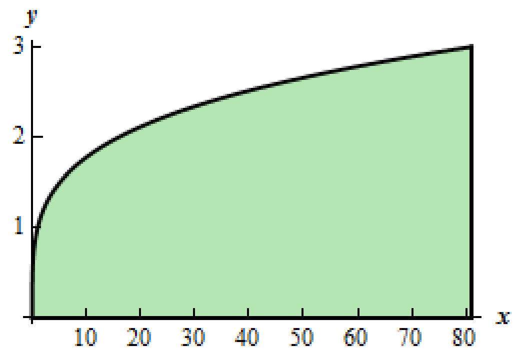
Now let's do the integral(s) for each part.

(a) $\displaystyle\iint_D 24y^2\,dA = \int_0^6\int_{\frac{1}{3}x}^{8-x} 24y^2\,dy\,dx = \int_0^6 8(8-x)^3-\tfrac{8}{27}x^3\,dx = \boxed{8064}$

(b) We'll need to do two integrals here and we should end up with the same final answer as **(a)**.

$$\iint_D 24y^2\,dA = \int_0^2\int_0^{3y} 24y^2\,dx\,dy + \int_2^8\int_0^{8-y} 24y^2\,dx\,dy$$
$$= \int_0^2 72y^3\,dy + \int_2^8 24(8-y)y^2\,dy = 288 + 7776 = \boxed{8064}$$

10. The region is sketched to the right. Here are the original as well as the reversed limits.



Original : $y^4 \le x \le 81 \qquad 0 \le y \le 3$

Reversed : $0 \le y \le \sqrt[4]{x} \qquad 0 \le x \le 81$

Here is the integral,

$$\int_0^3\int_{y^4}^{81} y^{11}\left(1+x^4\right)^{\frac{3}{2}} dx\,dy = \int_0^{81}\int_0^{\sqrt[4]{x}} y^{11}\left(1+x^4\right)^{\frac{3}{2}} dy\,dx$$
$$= \int_0^{81} \tfrac{1}{12}x^3\left(1+x^4\right)^{\frac{3}{2}} dx = \boxed{\tfrac{1}{120}\left(43046722^{\frac{5}{2}} - 1\right)}$$