1. (2 pts) We've got a portion of a disk to the right of the *y*-axis so *D* is : $2 \le r \le 6,\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$.

$$\iint_D 48xy^3\,dA = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_2^6 48\left(r\cos\theta\right)\left(r^3\sin^3\theta\right)r\,dr\,d\theta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_2^6 48r^5\cos\theta\sin^3\theta\,dr\,d\theta$$

$$= 372736\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos\theta\sin^3\theta\,d\theta = 93184\sin^4\theta\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \boxed{0}$$

9. (2 pts) We'll need the intersection of these two surfaces to determine *D*.

$$-4 = 8 - 3x^2 - 3z^2 \qquad \Rightarrow \qquad x^2 + z^2 = 4$$

So, *D* is the circle of radius 4 in the *xz*-plane. The limits on *y* are $-4 \le y \le 8 - 3x^2 - 3z^2$ and here is the first integration.

$$\iiint_E \sqrt{2x^2+2z^2}\,dV = \iint_D\int_{-4}^{8-3x^2-3z^2}\sqrt{2x^2+2z^2}\,dy\,dA = \iint_D\left(12-3x^2-3z^2\right)\sqrt{2x^2+2z^2}\,dA$$

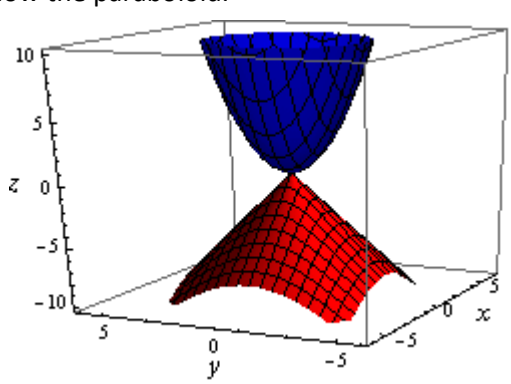
Now, the double integral looks like it would be best done in polar coordinates so we'll use them in the form : $x = r\cos\theta,\ z = r\sin\theta,\ x^2+z^2 = r^2$. The limits are : $0 \le r \le 2,\ 0 \le \theta \le 2\pi$.

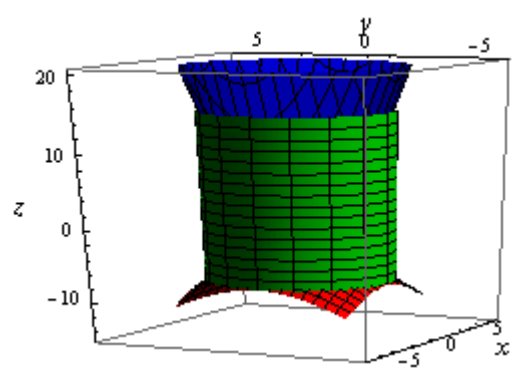
$$\iiint_E \sqrt{2x^2+2z^2}\,dV = \iint_D\left(12-3r^2\right)\sqrt{2}\,r\,dA = \int_0^{2\pi}\int_0^2\sqrt{2}\left(12r-3r^3\right)r\,dr\,d\theta = \int_0^{2\pi}\frac{64\sqrt{2}}{5}\,d\theta = \boxed{\frac{128\sqrt{2}\,\pi}{5}}$$

10. (3 pts) This is essentially the same integral that we did in 8 except we'll be integrating a different function.

$$V = \iiint_E dV = \int_0^2\int_0^{2y}\int_{3-\frac{3}{4}y-\frac{1}{2}z}^{21-3y-3z} dx\,dz\,dy = \int_0^2\int_0^{2y} 18 - \tfrac{9}{4}y - \tfrac{5}{2}z\,dz\,dy = \int_0^2 36y - \tfrac{19}{2}y^2\,dy = \boxed{\frac{140}{3}}$$

11. (3 pts) Here are a couple of figures to help us see the region. The lower surface is a cone and the upper surface is a paraboloid as we can see in the left figure. The walls of the region are the cylinder as shown in the figure to the right. So the region is the portion that is inside the cylinder, above the cone and below the paraboloid.





The limits for the integral are : $-2r \le z \le 8r^2,\ \ 0 \le r \le 4,\ \ 0 \le \theta \le 2\pi$ and the integral is,

$$\iiint_E x^2\,dV = \int_0^{2\pi}\int_0^4\int_{-2r}^{8r^2}\left(r\cos\theta\right)^2 r\,dz\,dr\,d\theta = \int_0^{2\pi}\int_0^4\left(8r^2+2r\right)r^3\cos^2\theta dr\,d\theta$$
$$= \int_0^{2\pi}\tfrac{88064}{15}\cos^2\theta\,d\theta = \int_0^{2\pi}\tfrac{44032}{15}\left(1+\cos\left(2\theta\right)\right)d\theta = \boxed{\tfrac{88064\pi}{15}}$$

Not Graded

2. In this case *D* is : $0 \le r \le 5,\ 0 \le \theta \le 2\pi$

$$\iint_D \mathbf{e}^{4x^2+4y^2-3}\,dA = \int_0^{2\pi}\int_0^5 r\mathbf{e}^{4r^2-3}dr\,d\theta = \int_0^{2\pi}\tfrac{1}{8}\mathbf{e}^{4r^2-3}\Big|_0^5 d\theta = \int_0^{2\pi}\tfrac{1}{8}\left(\mathbf{e}^{97}-\mathbf{e}^{-3}\right)d\theta = \boxed{\tfrac{\pi}{4}\left(\mathbf{e}^{97}-\mathbf{e}^{-3}\right)}$$

3. We have two paraboloids centered on the *x*-axis. The first one starts at 15 and opens backwards and the second starts at 1 and opens towards the front. They will intersect at,

$$15-5y^2-5z^2 = 2y^2+2z^2+1 \quad\Rightarrow\quad 7y^2+7z^2=14 \quad\Rightarrow\quad y^2+z^2=2$$

So, *D* is a disk of radius $\sqrt{2}$ centered at the origin. Now, we'll need to use the following "version" of polar coordinates : $y = r\sin\theta,\ z = r\cos\theta$. Using this means that $y^2+z^2=r^2$ and so the integral for the volume will then be,

$$V = \iint_D 15-5y^2-5z^2-\left(2y^2+2z^2+1\right)dA = \iint_D 14-7y^2-7z^2\,dA$$
$$= \int_0^{2\pi}\int_0^{\sqrt{2}}\left(14-7r^2\right)r\,dr\,d\theta = \int_0^{2\pi}\left(7r^2-\tfrac{7}{4}r^4\right)\Big|_0^{\sqrt{2}} d\theta = \int_0^{2\pi}7\,d\theta = \boxed{14\pi}$$

4. Not much to this problem.

$$A = \iint_D dA = \int_0^{2\pi}\int_0^a r\,dr\,d\theta = \tfrac{1}{2}\int_0^{2\pi} r^2\Big|_0^a d\theta = \tfrac{1}{2}\int_0^{2\pi} a^2\,d\theta = \pi a^2$$

5. Let's first get the limits for *x* and *y* : $0 \le x \le 4,\ -\sqrt{16-x^2} \le y \le 0$. The limits on *y* tell us that we have at most the lower portion of a circle of radius 4 centered at the origin and the limits on *x* tell us that it will in fact be the portion of the circle that is in the 4th quadrant and so the limits for *D* in terms of polar coordinates will be : $0 \le r \le 4,\ \tfrac{3\pi}{2} \le \theta \le 2\pi$. Note that you could just as easily use $-\tfrac{\pi}{2} \le \theta \le 0$ for the θ limits if you wanted to. The integral is then (don't forget that for the differential we have $dydx = dA = rdrd\theta$),

$$\int_0^4\int_{-\sqrt{16-x^2}}^0 \sin\left(1-x^2-y^2\right)dy\,dx = \int_{\frac{3\pi}{2}}^{2\pi}\int_0^4 r\cos\left(1-r^2\right)dr\,d\theta = \int_{\frac{3\pi}{2}}^{2\pi} -\tfrac{1}{2}\sin\left(1-r^2\right)\Big|_0^4 d\theta$$
$$= \int_{\frac{\pi}{2}}^{\pi}\tfrac{1}{2}\left(\sin\left(1\right)-\sin\left(-15\right)\right)d\theta = \boxed{\tfrac{\pi}{4}\left(\sin\left(1\right)+\sin\left(15\right)\right)}$$

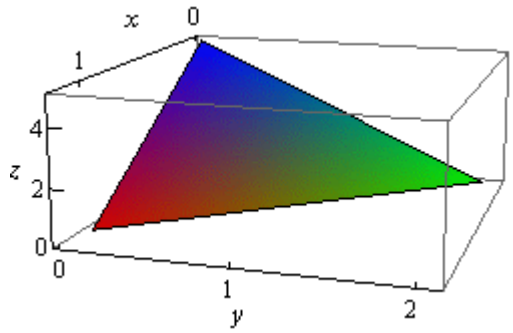
Note that sine is an odd function and so $\sin(-15) = -\sin(15)$!

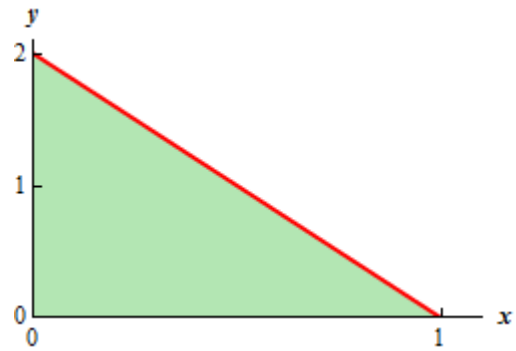
6. Not much to this problem.

$$\int_1^3 \int_{\sqrt{z}}^0 \int_0^{-2} y^2 x^3 \mathbf{e}^{z^3}\, dy\, dx\, dz = -\tfrac{8}{3}\int_1^3 \int_{\sqrt{z}}^0 x^3 \mathbf{e}^{z^3}\, dx\, dz = -\tfrac{8}{3}\int_1^3 \tfrac{1}{4} x^4 \mathbf{e}^{z^3}\Big|_{\sqrt{z}}^0 dz$$

$$= \tfrac{2}{3}\int_1^3 z^2 \mathbf{e}^{z^3}\, dz = \boxed{\tfrac{2}{9}\left(\mathbf{e}^{27} - \mathbf{e}\right)}$$

7. Here's a sketch of the plane and the region *D* in the *xy*-plane.



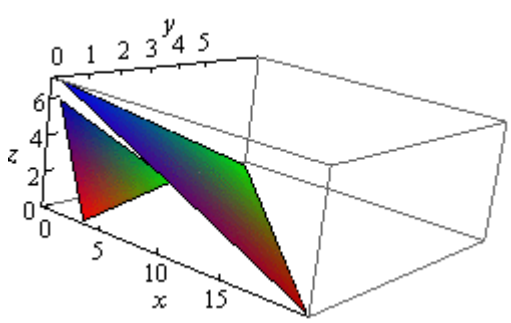


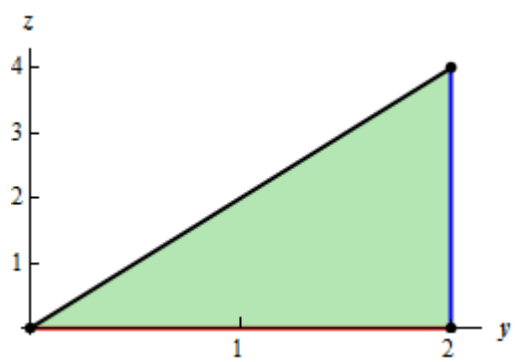
The equation for the line in the *xy*-plane is $10x + 5y = 10$ which we get by setting $z = 0$ in the equation of the plane. The limits for *E* are then : $0 \le x \le 1,\ 0 \le y \le 2 - 2x,\ 0 \le z \le 5 - 5x - \frac{5}{2}y$. The integral is then,

$$\iiint_E 32z\, dV = \int_0^1 \int_0^{2-2x} \int_0^{5-5x-\frac{5}{2}y} 32z\, dz\, dy\, dx = \int_0^1 \int_0^{2-2x} 16\left(5 - 5x - \tfrac{5}{2}y\right)^2 dy\, dx$$

$$= \int_0^1 -\tfrac{32}{15}\left(5 - 5x - \tfrac{5}{2}y\right)^3\Big|_0^{2-2x} dx = \int_0^1 -\tfrac{32}{15}\left(5 - 5x\right)^3 dx = \boxed{\frac{200}{3}}$$

8. Here's a sketch of the two planes as well as the region in *yz*-plane. Note as well that in order to "see" the two planes I've got the axis system "backwards" from what I usually have.





Because the region *D* is in the *yz*-plane we'll need the equations of the planes in terms of *x*.

Also, we can integrate *y* first and then *z* or the other way around. Note however that if we integrate *y* first one of the limits for each set will be zero and so that's the way we'll go.

$$0 \le y \le 2, \quad 0 \le z \le 2y, \quad 3 - \tfrac{3}{4}y - \tfrac{1}{2}z \le x \le 21 - 3y - 3z$$

The integral is then,

$$\iiint_E 36y\,dV = \int_0^2 \int_0^{2y} \int_{3-\frac{3}{4}y-\frac{1}{2}z}^{21-3y-3z} 36y\,dx\,dz\,dy = \int_0^2 \int_0^{2y} 36y\left(18 - \tfrac{9}{4}y - \tfrac{5}{2}z\right)dz\,dy$$

$$= 9\int_0^2 \int_0^{2y} 72y - 9y^2 - 10zy\,dz\,dy = 9\int_0^2 144y^2 - 38y^3\,dy = \boxed{2088}$$

12. We first need the intersection of the paraboloids.

$$3x^2 + 3z^2 - 24 = 12 - x^2 - z^2 \qquad \Rightarrow \qquad x^2 + z^2 = 9$$

So, we'll need to use the polar coordinates : $x = r\cos\theta,\ z = r\sin\theta$ and the limits will be,

$$3r^2 - 24 \le y \le 12 - r^2,\ 0 \le r \le 3,\ 0 \le \theta \le 2\pi$$

The volume is then,

$$V = \iiint_E dV = \int_0^{2\pi} \int_0^3 \int_{3r^2-24}^{12-r^2} r\,dz\,dr\,d\theta = \int_0^{2\pi} \int_0^3 18r^2 - r^4 dr\,d\theta = \int_0^{2\pi} 81\,d\theta = \boxed{162\pi}$$

13. This one isn't too bad once you think about it. Since we are inside the cylinder of radius *a* we know that the limits on *r* and θ will be,

$$0 \le \theta \le 2\pi \qquad\qquad 0 \le r \le a$$

Now, the height of *h* just tells us that we want the portion of the cylinder between the planes $z = 0$ and $z = h$. Therefore, the limits on *z* will be,

$$0 \le z \le h$$

The volume of the cylinder, using a triple integral is then,

$$V = \iiint_E dV = \int_0^{2\pi} \int_0^a \int_0^h r\,dz\,dr\,d\theta = \int_0^{2\pi} \int_1^{\sqrt{5}} r\,h\,dr\,d\theta = \int_0^{2\pi} \tfrac{1}{2}a^2\,h\,d\theta = \pi a^2\,h$$