Math 2415 **Homework Set 7** **10 Points**

Triple Integrals with Spherical Coordinates

For problems 1 and 2 you must use spherical coordinates to do the problems.

1. $\iiint\limits_E x\,z\,dV$ where *E* is the region below the sphere of radius 3 and inside the cone $\varphi = \frac{\pi}{3}$.

2. $\iiint\limits_E \left(x^2 + y^2 + z^2\right)^{\frac{3}{2}} dV$ where *E* is the region that lies in the first octant and between the spheres of radius 1 and 7.

3. Evaluate the following integral by first converting to spherical coordinates.

$$\int_{-3}^{0} \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} \int_{-\sqrt{27-x^2-y^2}}^{-\sqrt{2x^2+2y^2}} xy\,dz\,dx\,dy$$

Change of Variables

For problems 4 and 5 find the Jacobian of the transformation.

4. $x = 12uv^3 - u^2, \quad y = 8u + 12v^2$

5. $x = \mu\cos\alpha, \;\; y = \mu\sin\alpha$

For problems 6 – 8 find and graph the image of the set *R* under the given transformation. Note that the point of these problems is not necessarily to transform *R* into a "nice" region. Instead all we're trying to do is apply some transformations to regions.

6. *R* is the triangle with vertices (0, 0), (4, -2), (4, 12) and the transformation is $x = \frac{1}{3}u, \; y = \sqrt{1+2v}$.

Note that for each side you'll need the range of possible *y* values to in order to get a range of possible *u* values.

7. *R* is the rectangle bounded by the lines $x = 0, \; x = 2, \; y = 0, \; y = 8$ and the transformation is $u = y^2 - 2x^2, \; v = \frac{1}{2}xy$.

Note that sometimes transformations are giving "backwards" as they are here. In these cases it is usually best to plug the equations defining *R* into the transformations rather than plugging the transformations in the equations defining *R* as we've done to this point.

8. *R* is the disk given by $x^2 + y^2 \le 1$ and the transformation is $u = ax, \; v = by$.

9. If *R* is the parallelogram with vertices (-2, 6), (2, 4), (4, -2) and (0, 0) use the transformation $x = 4u - v$ $y = 3v - 2u$ to convert the region and evaluate the integral $\iint\limits_R 8x + 2y\,dA$.

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10. If *R* is the ellipse $\frac{x^2}{25}+\frac{y^2}{4}=1$ determine a transformation that will convert this into a circle of radius 1 and use that transformation to evaluate $\iint\limits_R 24y^2\,dA$.

Note that you've already seen how to turn a circle of radius 1 into an ellipse so use that as a guide to determine this transformation.

<u>Surface Area</u>

For problems 11 and 12 find the area of the given surface.

11. The part of the plane $6x+3y+8z=24$ that lies in the first octant.

12. The part of the surface $z=6+x+2y^2$ that lies above the triangle in the *xy*-plane with vertices (0,0), (8,2) and (0,2).

13. In class I gave you the formula for the surface area of $z=f\left(x,y\right)$ that lies above a region *D* in the *xy*-plane. Use a modification of this formula to find the surface area of the portion of $x=4y^2+4z^2-9$ that lies in behind of $x=3$.