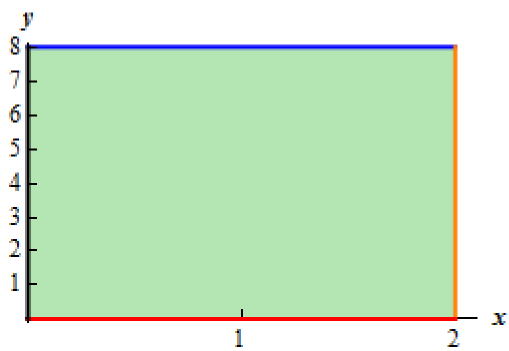
2. (2 pts) Here are the limits for this integral $1 \le \rho \le 4,\ 0 \le \theta \le \frac{\pi}{2},\ 0 \le \varphi \le \frac{\pi}{2}$ and the integral is,

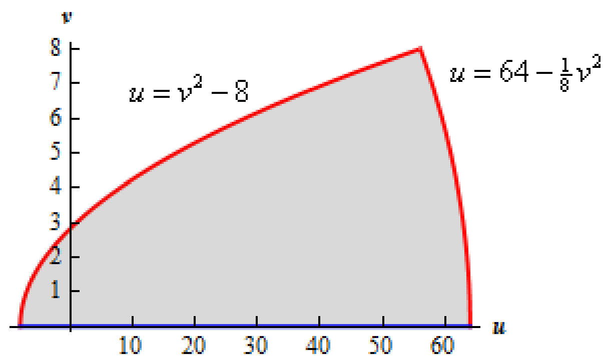
$$\iiint_E \left(x^2+y^2+z^2\right)^{\frac{3}{2}} dV = \int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\int_1^7 \rho^3\rho^2 \sin\varphi \, d\rho \, d\theta \, d\varphi$$

$$= \int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\int_1^7 \rho^5 \sin\varphi \, d\rho \, d\theta \, d\varphi = \int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} 19608 \sin\varphi \, d\theta \, d\varphi$$

$$= \int_0^{\frac{\pi}{2}} 9804\pi \sin\varphi \, d\varphi = \boxed{9804\pi}$$

7. (4 pts) Here is a sketch of the original region and the transformed region. Transformation work is below.





Here's the transformation work and as noted we'll be plugging the equations for the sides of the rectangle into the transformations and see what they tell us.

$x = 0,\ 0 \le y \le 8$

$v = 0, \quad u = y^2 \qquad \underline{v = 0},\ 0 \le y \le 8 \to 0 \le y^2 \le 64 \to \underline{0 \le u \le 64}$

$y = 0,\ 0 \le x \le 2$

$v = 0, \quad u = -2x^2 \qquad \underline{v = 0},\ 0 \le x \le 2 \to -8 \le -2x^2 \le 0 \to \underline{-8 \le u \le 0}$

Note that all these two tell us is that these two sides collectively transform into the single side $v = 0,\ -8 \le u \le 64$.

$x = 2,\ 0 \le y \le 8$

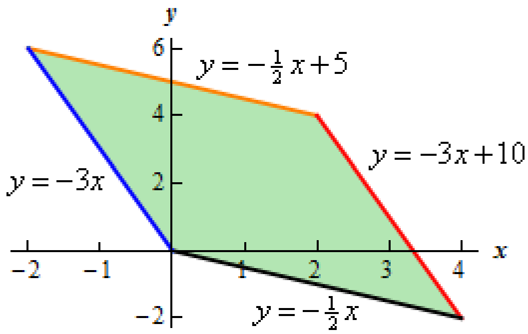
$v = y, \quad u = y^2 - 8 = v^2 - 8 \qquad \underline{u = v^2 - 8},\ 0 \le y \le 8 \to \underline{0 \le v \le 8}$

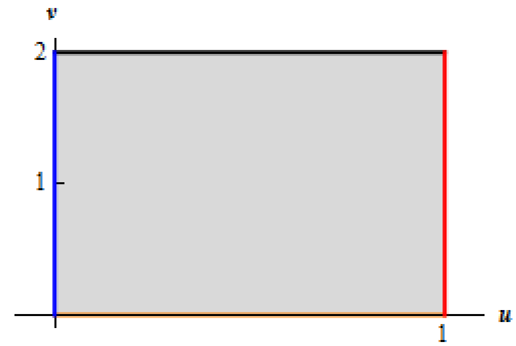
$y = 8,\ 0 \le x \le 2$

$v = 4x, \quad u = 64 - 2x^2 = 64 - \frac{v^2}{8} \qquad \underline{u = 64 - \frac{v^2}{8}},\ 0 \le 4x \le 6 \to \underline{0 \le v \le 8}$

Note that the two curves that form the "top" of the new region meet at $(56, 8)$.

9. (4 pts) Here is a sketch of the original region and transformed region. Transformation work is below.





Here's the transformation work.

$y = -\frac{1}{2}x$	$3v - 2u = -\frac{1}{2}(4u - v)$	$\Rightarrow$	$\underline{v = 0}$
$y = -\frac{1}{2}x + 5$	$3v - 2u = -\frac{1}{2}(4u - v) + 5$	$\Rightarrow$	$\underline{v = 2}$
$y = -3x$	$3v - 2u = -3(4u - v)$	$\Rightarrow$	$\underline{u = 0}$
$y = -3x + 10$	$3v - 2u = -3(4u - v) + 10$	$\Rightarrow$	$\underline{u = 1}$

Next we'll need the Jacobian.

$$\begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ -2 & 3 \end{vmatrix} = 12 - 2 = 10$$

The integral is then,

$$\iint_R 8x + 2y\,dA = \iint_S \left[8(4u - v) + 2(3v - 2u)\right]\left|10\right|dA = 10\int_0^2\int_0^1 28u - 2v\,du\,dv$$

$$= 10\int_0^2 14 - 2v\,dv = \boxed{240}$$

Not Graded

1. Here are the limits for this integral $0 \le \rho \le 3,\ 0 \le \theta \le 2\pi,\ 0 \le \varphi \le \frac{\pi}{3}$ and the integral is,

$$\iiint_E x\,z\,dV = \int_0^{\frac{\pi}{3}}\int_0^{2\pi}\int_0^3 (\rho\sin\varphi\cos\theta)(\rho\cos\varphi)\rho^2\sin\varphi\,d\rho\,d\theta\,d\varphi$$

$$= \int_0^{\frac{\pi}{3}}\int_0^{2\pi}\int_0^3 \rho^4\cos\varphi\sin\varphi\cos\theta\,d\rho\,d\theta\,d\varphi = \int_0^{\frac{\pi}{3}}\int_0^{2\pi} \tfrac{243}{5}\cos\varphi\sin\varphi\cos\theta\,d\theta\,d\varphi$$

$$= \int_0^{\frac{\pi}{6}} 0\,d\varphi = \boxed{0}$$

3. First let's get the limits on *x*, *y*, and *z*.

$$-3 \le y \le 0,\ -\sqrt{9-y^2} \le x \le \sqrt{9-y^2},\ -\sqrt{27-x^2-y^2} \le z \le -\sqrt{2x^2+2y^2}$$

The limits on *x* and *y* tell us that the region *D* in the original integral was the lower half of the circle of radius 3 centered at the origin. The lower surface of the solid *E* is a sphere of radius $\sqrt{27} = 3\sqrt{3}$ while the upper surface of *E* is a cone. From this information we can get the following two sets of limits.

$$0 \le \rho \le 3\sqrt{3}, \quad \pi \le \theta \le 2\pi$$

Now, we just need to find the limits on φ and we can get that from the cone.

$$z = -\sqrt{2x^2+2y^2} = -\sqrt{2}\sqrt{x^2+y^2}$$

$$\rho\cos\varphi = -\sqrt{2}\rho\sin\varphi$$

$$-\tfrac{1}{\sqrt{2}} = \tan\varphi \qquad \Rightarrow \qquad \varphi = 2.5261$$

Recall that the φ must be in the range $0 \le \varphi \le \pi$. Finally because this is a cone below the *xy*-plane the limits on φ must be $2.2561 \le \varphi \le \pi$. The integral is then,

$$\int_{-3}^{0}\int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}}\int_{-\sqrt{27-x^2-y^2}}^{-\sqrt{2x^2+2y^2}} xy\,dz\,dx\,dy = \int_{2.5261}^{\pi}\int_{\pi}^{2\pi}\int_{0}^{3\sqrt{3}} (\rho\sin\varphi\cos\theta)(\rho\sin\varphi\sin\theta)\rho^2\sin\varphi\,d\rho\,d\theta\,d\varphi$$

$$= \int_{2.5261}^{\pi}\int_{\pi}^{2\pi}\int_{0}^{3\sqrt{3}} \rho^4\sin^3\varphi\cos\theta\sin\theta\,d\rho\,d\theta\,d\varphi$$

$$= \int_{2.5261}^{\pi}\int_{\pi}^{2\pi} \tfrac{2187\sqrt{3}}{5}\sin^3\varphi\cos\theta\sin\theta\,d\theta\,d\varphi$$

$$= \int_{2.5261}^{\pi}\int_{\pi}^{2\pi} \tfrac{2187\sqrt{3}}{10}\sin^3\varphi\sin(2\theta)\,d\theta\,d\varphi = \int_{2.5261}^{\pi} 0\,d\varphi = \boxed{0}$$

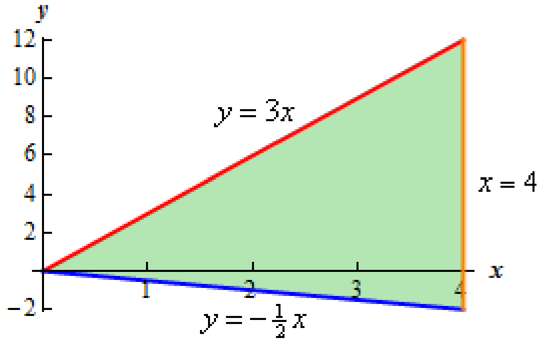
4.

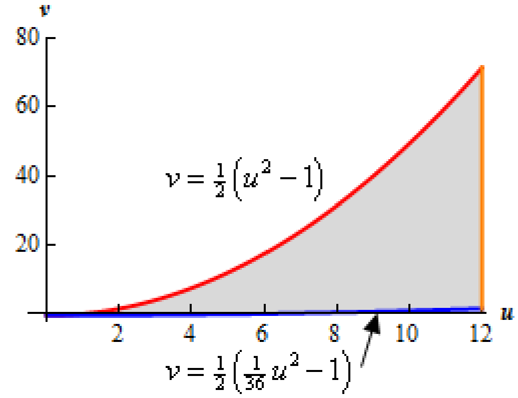
$$\begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 12v^3 - 2u & 36uv^2 \\ 8 & 24v \end{vmatrix} = 24v(12v^3-2u) - 288uv^2 = \boxed{288v^4 - 48uv - 288uv^2}$$

5.

$$\begin{vmatrix} \frac{\partial x}{\partial \mu} & \frac{\partial x}{\partial \alpha} \\ \frac{\partial y}{\partial \mu} & \frac{\partial y}{\partial \alpha} \end{vmatrix} = \begin{vmatrix} \cos\alpha & -\mu\sin\alpha \\ \sin\theta & \mu\cos\alpha \end{vmatrix} = \mu\cos^2\alpha + \mu\sin^2\alpha = \boxed{\mu}$$

6. Here is a sketch of the original region and the transformed region. Note that due to the scale of things the bottom curve is a little difficult to see. Transformation work is below.





Here is the work for transforming each of the sides.

$y=-\frac{1}{2}x,\ \ 0\le x\le 4,\ \ -2\le y\le 0$

$$\sqrt{1+2v}=-\tfrac{1}{6}u \quad\to\quad \underline{v=\tfrac{1}{2}\left(\tfrac{1}{36}u^2-1\right)} \qquad\qquad 0\le x\le 4 \to 0\le \tfrac{1}{3}u\le 4 \quad\to\quad \underline{0\le u\le 12}$$

Note that the range of *u*'s that I got from the range of *x*'s is not really consistent with the original *uv* equation (which pretty much required that *u* be negative). The transformation really does force *u* to be positive since *u* and *x* clearly need to have the same sign in order to "satisfy" the transformation. This means that the "proper" curve really is the right portion of the parabola rather than the left portion as implied by the initial equation transformation.

Basically, this was just a poorly written problem on my part and the blue line above should have been $y=\frac{1}{2}x$ (*i.e.* positive, to avoid this issue)!

$y=3x,\ \ 0\le x\le 4,\ \ 0\le y\le 12$

$$\sqrt{1+2v}=u \quad\to\quad \underline{v=\tfrac{1}{2}\left(u^2-1\right)} \qquad\qquad 0\le x\le 4 \quad\to\quad 0\le \tfrac{1}{3}u\le 4 \quad\to\quad \underline{0\le u\le 12}$$

$x=4,\ \ -2\le y\le 12$

$$4=\tfrac{1}{3}u \quad\Rightarrow\quad \underline{u=12} \qquad\qquad -2\le y\le 12 \quad\to\quad -2\le\sqrt{1+2v}\le 4 \quad\to\quad \underline{\tfrac{3}{2}\le v\le \tfrac{143}{2}}$$

8. Not much to do here other than solve the transforms for *x* and *y* and plug them into the equation.

$$\left(\frac{u}{a}\right)^2+\left(\frac{v}{b}\right)^2\le 1 \qquad\qquad\Rightarrow\qquad\qquad \frac{u^2}{a^2}+\frac{v^2}{b^2}\le 1$$

The new region is then an ellipse with vertices $(-a,0),\ (a,0),\ (0,b),\ (0,-b)$.

10. We saw in 8 how to turn a disk into an ellipse and so it shouldn't be too surprising that the transformation is : $x = 5u,\;\; y = 2v$. Using this transformation we get,

$$\frac{(5u)^2}{25} + \frac{(2v)^2}{4} = u^2 + v^2 = 1$$

The Jacobian is,

$$\begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 5 & 0 \\ 0 & 2 \end{vmatrix} = 10$$

The integral is then,

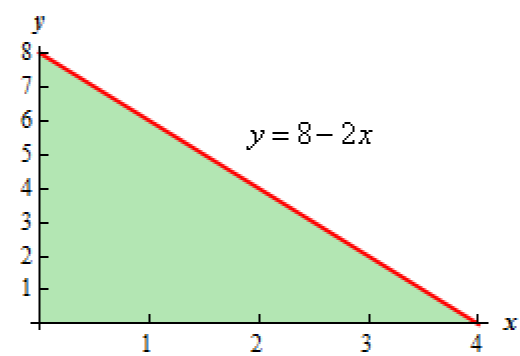
$$\iint_R 24y^2\,dA = \iint_S 24(2v)^2\,|10|\,dA = 960\int_0^{2\pi}\int_0^1 r^3\sin^2\theta\,dr\,d\theta$$
$$= 240\int_0^{2\pi}\sin^2\theta\,d\theta = 120\int_0^{2\pi}\left(1-\cos(2\theta)\right)d\theta = \boxed{240\pi}$$

11. A sketch of the region *D* is to the right and the limits here are,

$$0 \le x \le 4 \quad 0 \le y \le 8 - 2x$$

The function is $z = 3 - \frac{4}{3}x - \frac{3}{8}y$ and the surface area is,

$$A = \iint_D \sqrt{\tfrac{16}{9} + \tfrac{9}{64} + 1}\,dA = \tfrac{41}{24}\int_0^4\int_0^{8-2x} dy\,dx$$
$$= \tfrac{41}{24}\int_0^4 8 - 2x\,dx = \boxed{\tfrac{82}{3}}$$

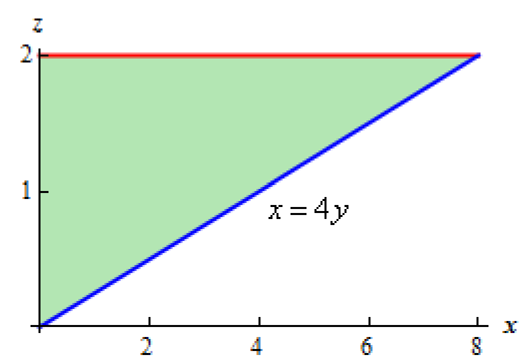


12. A sketch of the region *D* is to the right. For reasons that will be apparent once we do the integral we'll use the following limits

$$0 \le y \le 2 \quad 0 \le x \le 4y$$

The area is then,

$$A = \iint_D \sqrt{2 + 4y^2}\,dA = \int_0^2\int_0^{4y}\sqrt{2+4y^2}\,dx\,dy$$
$$= \int_0^2 4y\sqrt{2+4y^2}\,dy = \boxed{\tfrac{1}{3}\left(18^{\frac{3}{2}} - 2^{\frac{3}{2}}\right)}$$



13. The formula we'll need to use here for $x = f(y,z)$ is,

$$S = \iint_D \sqrt{1 + \left[f_y\right]^2 + \left[f_z\right]^2}\,dA$$

In this case the region *D* will be the intersection of these two surfaces.

$$3 = 4y^2 + 4z^2 - 9 \qquad \Rightarrow \qquad y^2 + z^2 = 3$$

So, we'll need to use the "polar" coordinates $y = r\sin\theta,\ z = r\cos\theta$. The surface area is,

$$S = \iint\limits_D \sqrt{1+64y^2+64z^2}\,dA = \int_0^{2\pi}\int_0^{\sqrt{3}} r\sqrt{1+64r^2}\,dr\,d\theta = \int_0^{2\pi} \tfrac{1}{192}\left(193^{\frac{3}{2}}-1\right)d\theta = \boxed{\tfrac{\pi}{96}\left(193^{\frac{3}{2}}-1\right)}$$