**1. (2 pts)** $\boxed{\nabla f(x,y) = \left\langle -2x\left(6y - y^3\right)\mathbf{e}^{2-x^2}, \left(6 - 3y^2\right)\mathbf{e}^{2-x^2}\right\rangle}$

**6. (2 pts)** This is going "backwards" in *x* so we need to use the vector form of the line segment starting at (-1, 10) and ending at (-19, 10).

$$\vec{r}(t) = (1-t)\langle -1,10\rangle + t\langle -19,10\rangle = \langle -1-18t, 10\rangle \qquad 0 \le t \le 1$$

**8. (2 pts)** The parametric equations for the curve, with clockwise orientation, is

$$\vec{r}(t) = \langle -2\cos t, 2\sin t\rangle,\ 0 \le t \le \pi$$

The integral is,

$$\int_C x\,y^2\,ds = \int_0^\pi (-2\cos t)\left(4\sin^2 t\right)\sqrt{(-2\sin t)^2 + (-2\cos t)^2}\,dt = -16\int_0^\pi \cos t \sin^2 t\,dt$$

$$= -\tfrac{16}{3}\sin^3(t)\Big|_0^\pi = 0$$

**11. (2 pts)** Here are the parametric equations for each curve.

$$C_1 :\ \vec{r}(t) = \langle 4-4t, 3\rangle,\ 0 \le t \le 1$$

$$C_2 :\ \vec{r}(t) = \langle 3\cos t, 3\sin t\rangle,\ \tfrac{\pi}{2} \le t \le \tfrac{3\pi}{2}$$

Here is the integral for each curve.

$$\int_{C_1} xy\,dx + x^2dy = \int_0^1 (4-4t)(3)(-4) + (4-4t)^2(0)\,dt = 12\int_0^1 4-4t\,dt = \underline{-24}$$

$$\int_{C_2} xy\,dx + x^2dy = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (3\cos t)(3\sin t)(-3\sin t) + (3\cos t)^2(3\cos t)\,dt$$

$$= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} -27\cos t\sin^2 t + 27\cos^3 t\,dt$$

$$= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} -27\cos t\sin^2 t + 27\cos t\left(1 - \sin^2 t\right)dt$$

$$= \left(-9\sin^3 t + 27\left(\sin t - \tfrac{1}{3}\sin^3 t\right)\right)\Big|_{\frac{\pi}{2}}^{\frac{3\pi}{2}} = \underline{-18}$$

$$\int_C xy\,dx + x^2dy = \int_{C_1} xy\,dx + x^2dy + \int_{C_2} xy\,dx + x^2dy = -24 - 18 = \boxed{-42}$$

**12. (2 pts)** Here's the parameterization of the line : $\vec{r}\left(t\right)=\left\langle 2+3t,t,-1-t\right\rangle,\ 0\le t\le 1$. Here's the integral,

$$\int_C \left(4x-y\right)dx-z^2dy-\left(x+y-4z\right)dz=\int_0^1\left(8+11t\right)\left(3\right)-\left(-1-t\right)^2\left(1\right)-\left(8t+6\right)\left(-1\right)dt$$

$$=\int_0^1 -t^2+39t+29\,dt=\boxed{\tfrac{289}{6}}$$

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***Not Graded***

**2.**

$$\boxed{\nabla f\left(x,y,z\right)=\left\langle 7z^5\cos\left(y^2-x^2\right)+14x^2z^5\sin\left(y^2-x^2\right),-14xyz^5\sin\left(y^2-x^2\right),35xz^4\cos\left(y^2-x^2\right)\right\rangle}$$

**3.** $\vec{r}\left(t\right)=\left\langle \frac{1}{3}\cos t,4\sin t\right\rangle$ or $x=\frac{1}{3}\cos t,\ y=4\sin t \qquad 0\le t\le 2\pi$

**4.** $\vec{r}\left(y\right)=\left\langle y^5-y\sqrt{1+y},y\right\rangle$ or $x=y^5-y\sqrt{1+y},\ y=y \qquad 2\le y\le 12$

**5.** $\vec{r}\left(t\right)=\left(1-t\right)\left\langle -5,8\right\rangle+t\left\langle -2,-6\right\rangle=\left\langle -5+3t,8-14t\right\rangle \qquad 0\le t\le 1$

**7.**

**(a)** First we'll need the vector function for the line :

$$\vec{r}\left(t\right)=\left(1-t\right)\left\langle 1,-1\right\rangle+t\left\langle -3,1\right\rangle=\left\langle 1-4t,-1+2t\right\rangle,\ 0\le t\le 1$$

The integral is then,

$$\int_C 8y\,ds=\int_0^1 8\left(-1+2t\right)\sqrt{\left(-4\right)^2+\left(2\right)^2}\,dt=8\sqrt{20}\int_0^1 -8-16dt=\boxed{16\sqrt{5}\left(0\right)=0}$$

**(b)** Here are the parametric equations for each portion of the curve and following the given orientation as instructed to do by the problem statement.

$$C_1:\ \vec{r}\left(t\right)=\left\langle t,-1\right\rangle,\ 1\le t\le 5 \qquad \left(\text{line } y=-1\right)$$

$$C_2:\ \vec{r}\left(t\right)=\left\langle 6-t^2,t\right\rangle,\ -1\le t\le 3 \qquad \left(\text{curve } x=6-y^2\right)$$

$$C_3:\ \vec{r}\left(t\right)=\left\langle -3,3-2t\right\rangle,\ 0\le t\le 1 \qquad \left(\text{line } x=-3\right)$$

Here's the integral for each curve.

$$\int_{C_1} 8y\,ds=\int_1^5 8\left(-1\right)\sqrt{\left(1\right)^2+\left(0\right)^2}\,dt=\int_1^5 -8\,dt=\underline{-32}$$

$$\int_{C_2} 8y\,ds = \int_{-1}^{3} 8(t)\sqrt{(-2t)^2 + (1)^2}\,dt = \int_{-1}^{3} 8t\sqrt{4t^2+1}\,dt = \underline{\tfrac{2}{3}\left(37^{\frac{3}{2}} - 5^{\frac{3}{2}}\right)}$$

$$\int_{C_3} 8y\,ds = \int_{0}^{1} 8(3-2t)\sqrt{(0)^2 + (-2)^2}\,dt = 2\int_{0}^{1} 24 - 16t\,dt = \underline{32}$$

The overall integral is then,

$$\int_C 8y\,ds = \int_{C_1} 8y\,ds + \int_{C_2} 8y\,ds + \int_{C_3} 8y\,ds = \boxed{-32 + \tfrac{2}{3}\left(37^{\frac{3}{2}} - 5^{\frac{3}{2}}\right) + 32 = 142.588}$$

**9.** Not much to do here other than do the integral.

$$\int_C 4z + x^2 + 8y\,ds = \int_0^4 \left[4(-7) + (9t)^2 + 8(2+4t)\right]\sqrt{(9)^2 + (4)^2 + (0)^2}\,dt$$

$$= \sqrt{97}\int_0^4 81t^2 + 32t - 12\,dt = \boxed{1936\sqrt{97}}$$

**10.** Not much to do other than do the integral.

$$\int_C \cos(2+y)\,dy = \int_0^3 \cos\left(2+\sqrt{x+1}\right)\left(\tfrac{1}{2}(x+1)^{-\frac{1}{2}}\right)dx = \sin\left(2+\sqrt{x+1}\right)\Big|_0^3$$

$$= \boxed{\sin(4) - \sin(3) = -0.8979}$$