Line Integrals of Vector Fields

For problems 1 and 2 evaluate $\int_C \vec{F} \bullet d\vec{r}$ for the given vector field, $\vec{F}$, and the given curve, *C*.

1. $\vec{F}(x,y) = x^2\,\vec{i} - (y-x)\,\vec{j}$ where *C* is the right half of $\frac{x^2}{4} + \frac{y^2}{7} = 1$ and the direction of motion is from the bottom to top.

2. $\vec{F}(x,y,z) = \cos(z)\,\vec{i} - x^2y^3\,\vec{j} + (4z+x)\,\vec{k}$, *C* is given by $\vec{r}(t) = (1+t^2)\,\vec{i} - 2t\,\vec{j} + t^2\,\vec{k},\ -1 \le t \le 0$

Fundamental Theorem for Line Integrals

3. Evaluate $\int_C \nabla f \bullet d\vec{r}$ for $f(x,y,z) = \cos(4x)\sin(\pi y) - \frac{(x+1)z^2}{2y}$ and *C* is given by $\vec{r}(t) = \left\langle t^2 - 4t + 3, \tfrac{1}{2}t, 10 - t \right\rangle,\ 1 \le t \le 3$.

4. For the vector fields and curve in **#1** above is it possible to determine if $\int_C \vec{F} \bullet d\vec{r}$ is independent of path? If so, is the integral independent of path and if it is not possible explain why not.

Conservative Vector Fields

For problems 5 and 6 determine if the vector field, $\vec{F}$, is conservative or not. If it is conservative find the potential function for the vector field.

5. $\vec{F} = (6x^4 - 8y + x^2y^2)\,\vec{i} + (8x + x^2y^2)\,\vec{j}$

6. $\vec{F} = (y^4 + 18x^2y^2 - 2xy + 6)\,\vec{i} - (x^2 + 24y - 12x^3y - 4xy^3)\,\vec{j}$

For problems 7 – 9 find the potential function for the vector field and then evaluate $\int_C \vec{F} \bullet d\vec{r}$ for the given curve *C*.

7. $\vec{F} = (y^4 + 18x^2y^2 - 2xy + 6)\,\vec{i} - (x^2 + 24y - 12x^3y - 4xy^3)\,\vec{j}$ and *C* is the line segment from (1, -2) to (3, 0).

8. $\vec{F} = \left(4x^2\mathbf{e}^{4x-2y} + 2x\mathbf{e}^{4x-2y} - 4\mathbf{e}^{4x-2y}\right)\vec{i} + 2\left(\mathbf{e}^{4x-2y} - x^2\mathbf{e}^{4x-2y} - y\right)\vec{j}$ and *C* is the left half of the circle of radius 5 that is centered at (2,3) starting at the bottom and ending at the top.

Continued on Back $\Rightarrow$

9. $\vec{F} = \left(4 + \frac{y}{x} - 2xz^3\right)\vec{i} + \ln(xz)\,\vec{j} - \left(3x^2z^2 - \frac{y}{z}\right)\vec{k}$ and *C* is $\vec{r}(t) = 2\vec{i} - (2-3t)\,\vec{j} + (t^2+1)\,\vec{k}$,
$0 \le t \le 2$.

<u>Green's Theorem</u>

For problems 10 and 11 evaluate the line integral (a) directly and (b) using Green's Theorem. Assume all curves have the positive orientation.

10. $\oint_C (4x+y)\,dy - (8y-3)\,dx$ where *C* is the circle of radius 2 centered at the origin.

11. $\oint_C 4xy\,dx + (x^2+2y)\,dx$ where *C* is the triangle with vertices (0, 0), (2, 4) and (2, -2).