**Math 2415** **Homework Set 10** **10 Points**

**<u>Green's Theorem</u>**

For problems 1 and 2 sketch the positively oriented curve (clearly indicating the positive orientation) and use Green's Theorem to evaluate the line integral along the given curve.

**1.** $\oint_C 16xy\,dx + \left(1-x^2\right)dy$ where *C* is the portion of $y = x^2 + 4$ from (-1, 5) to (2, 8) and the three line segments : (*i*) from (2, 1) to (2, 8), (*ii*) from (-1, 1) to (2, 1), (*iii*) from (-1, 1) to (-1, 5) .

**2.** $\int_C \vec{F} \bullet d\vec{r}$ where $\vec{F} = \left(y^3 + y^2\right)\vec{i} - x^3\vec{j}$ and *C* is the portion of $x^2 + y^2 = 16$ from $\left(2, 2\sqrt{3}\right)$ to $\left(-2\sqrt{3}, 2\right)$ and the two line segments : (i) from (0,0) to $\left(2, 2\sqrt{3}\right)$ and (*ii*) (0,0) to $\left(-2\sqrt{3}, 2\right)$.

**<u>Curl and Diverence</u>**

**3.** Find the curl and divergence of $\vec{F} = \frac{y^2}{xz}\vec{i} + x\ln\left(x\right)\vec{j} + \left(4z + x^2 - y^3\right)\vec{k}$

For problems 4 and 5 use the curl to determine if the given vector field is conservative or not.

**4.** $\vec{F} = \frac{y^2}{xz}\vec{i} + x\ln\left(x\right)\vec{j} + \left(4z + x^2 - y^3\right)\vec{k}$

**5.** $\vec{F} = \left(4 + \frac{y}{x} - 2xz^3\right)\vec{i} + \ln\left(xz\right)\vec{j} - \left(3x^2z^2 - \frac{y}{z}\right)\vec{k}$

**<u>Parametric Surfaces</u>**

For problems 6 – 9 find a parametric representation for the given surface.

**6.** The plane containing the points (1, -1, 0), (2, 0, 3) and (1, 4, 1).

**7.** The portion of $z = 8x^2 + 8y^2 + 3$ that lies below $z = 9$.

**8.** The cylinder $x^2 + z^2 = 3$ between $y = -2$ and $y = 15$.

**9.** The lower half of the sphere $x^2 + y^2 + z^2 = 16$.

**10.** Find the tangent plane to $x = 7u - uv$, $y = 1 - u^2$, $z = v^2 - 4v$ at the point (-2, -3, 12).

For problems 11 and 12 find the area of the given surface.

**11.** The portion of $z = 1 + x^2 + 8y$ that lies above the triangle with vertices (0, 0), (-4, 0) and (-4, -2).

**12.** The surface $r\left(u,v\right) = v\,\vec{i} - 9uv\,\vec{j} - u\,\vec{k}$ where $u^2 + v^2 \le 4$.