Math 2415 **Homework Set 11** **10 Points**

Surface Integrals

For problems 1 – 3 evaluate the given surface integral.

1. $\iint\limits_S z + 3x\, dS$ where *S* is the portion of $6x + 2y + 2z = 12$ that lies in the first octant.

2. $\iint\limits_S 8z - 24\, dS$ where *S* is the portion of $z = x^2 + y^2 + 3$ that lies below $z = 4$.

3. $\iint\limits_S x - 2y\, dS$ where *S* is the portion of the cylinder $x^2 + z^2 = 9$ bounded by $y = -1$ and $y = x + 6$.

Surface Integrals of Vector Fields

For problems 4 and 5 evaluate $\iint\limits_S \vec{F} \bullet d\vec{S}$ for the given vector field and surface.

4. $\vec{F}(x, y, z) = 12x\,\vec{i} + z\,\vec{j} + (6 - y)\vec{k}$ and *S* is the portion of $x = 4 - y^2 - z^2$ that lies in front of $x = 3$ and oriented in the direction of the negative *x*-axis.

5. $\vec{F}(x, y, z) = z\,\vec{i} + (y - 6)\,\vec{j} - 3\vec{k}$ and *S* is the surface from problem **#3** with the positive orientation.

Stokes' Theorem

6. Use Stokes' Theorem to evaluate $\iint\limits_S \operatorname{curl} \vec{F} \bullet d\vec{S}$ where $\vec{F}(x, y, z) = y\,\vec{i} + (y^2 - z^3)\,\vec{j} - 12x\,\vec{k}$ and *S* is the portion of $y = 2x^2 + 2z^2$ that lies inside the cylinder $x^2 + z^2 = 4$ oriented in the direction of the positive *y*-axis

7. Use Stokes' Theorem to evaluate $\int\limits_C \vec{F} \bullet d\vec{r}$ where $\vec{F}(x, y, z) = 3y\,\vec{i} + x^3\,\vec{j} - (5z + 2)\vec{k}$ and *C* is the circle $x^2 + y^2 = 2$ at $z = -3$ and *C* is oriented in the clock-wise direction when viewed from above.

Hint : You'll need an easy to work with surface whose intersection with the plane $z = -3$ is the circle $x^2 + y^2 = 2$ that will also have the correct orientation. By this point in the semester you've worked many times with one particular kind of surface that will do this.

Divergence Theorem

8. Use the Divergence Theorem to evaluate $\iint\limits_S \vec{F} \bullet d\vec{r}$ where $\vec{F}(x, y, z) = 4xz\,\vec{i} + 2y\,\vec{j} - 4z\,\vec{k}$ and *S* is the portion of the surface bounded by the two hemispheres $z = \sqrt{4 - x^2 - y^2}$ and $z = \sqrt{9 - x^2 - y^2}$ that lies in the first octant.