Math 3301 **Homework Set 1** **10 Points**

Basics

Sketch the direction field for each of the following differential equations. Based on your direction field sketch determine the behavior of the solution, $y(t)$, as $t \to \infty$ (*i.e.* the long term behavior). If this behavior depends upon the value of $y(0)$ give this dependence.

1. $\frac{dy}{dt} = y(y-3)^2(y+1)$

2. $y' = (y-1)\left(1-\mathbf{e}^{3y+6}\right)$

Linear Differential Equations

For problems 3 & 4 solve the given IVP.

3. $\left(x^2+4\right)y' = x\sqrt{x^2+4}+2xy \qquad y(0)=8$

4. $2t\,y' + (3t-8)\,y = t^6\mathbf{e}^{-\frac{3}{2}t}\sin(\pi t) \qquad y(1)=0$

5. It is known that the solution to the following differential equation will have a relative minimum of $y=3$. Assuming that the solution and its derivative exist and are continuous for all *t* determine the value of *t*, $t=t_0$, for which the solution will have this value, *i.e.* $y(t_0)=3$. Note that because you don't have an initial condition you can't actually solve this differential equation. It is still possible however to answer this question.

$$y' + 6y = 4 + 7\mathbf{e}^{t}$$

Hint : Recall from Calc I where relative extrema may occur and don't forget the differential equation, Just because you can't solve the differential equation doesn't mean that it's not needed!

6. Find the solution to the following IVP in terms of y_0. Determine the value of y_0 for which the solution will have a relative minimum at $t=0.5$.

$$y' + 4y = t - \mathbf{e}^{-2t} \qquad y(0) = y_0$$

Hint : Once you have the solution in terms of y_0 you already know *where* the relative minimum is, if there was just a way (cough, cough, #5….) to determine *what* the relative minimum is the rest should be pretty easy.

7. Find the solution to the following IVP in terms of α . Find all possible long term behaviors of the solution at $t \to \infty$. If this behavior depends on the value of α give this dependence.

$$2y' - 6y = 6 - 7\mathbf{e}^{t} \qquad y(0) = 4 - \alpha^2$$