

Separable Differential Equations

For problems 1 & 2 find the solution to the given IVP and determine the interval of validity for the solution. Any approximate answers must be to at least the 4th decimal place and you may need computational aids in finding some of the intervals of validity.

1. $(1 + e^{2x})y' = y^4 e^{2x}$ $y(0) = -1$

2. $(6x - 7)y' = \frac{1}{y^2}$ $y(0) = 2$

3. Solve the following differential equation and determine the interval of validity. Any approximate answers must be to at least the 4th decimal place.

$$y' = \frac{1 - 8x}{5 + y} \qquad y(0) = -1$$

Modeling, Part I

For problems 4 & 5 you MUST set up and solve the appropriate IVP(s) in order to receive any credit for the problem. Any decimals must be to at least the 4th decimal place.

4. A 400 gallon tank contains 250 gallons of water with 25 ounces of salt dissolved in it. Salt water with a concentration of $c(t) = 12 - 7e^{-\frac{t}{25}}$ ounces/gal is flowing into the tank at a rate of 5 gallons/min and a well mixed solution flows out at a rate of 5 gallons/min. If left to forever, what would be the equilibrium (*i.e.* what would be the amount of salt in the tank as $t \rightarrow \infty$) amount of salt in the water?

5. A 1000 liter tank initial contains 800 liters of water with 100 grams of contaminate dissolved in it. Contaminated water with a concentration of 20 grams/liter flows into the tank at a rate of 8 liters/hr and a well mixed solution flows out at a rate of 2 liters/hr. This will continue until the tank overflows. Once the tank overflows the concentration of the incoming of contaminated water is decreased to 12 grams/liter and the incoming flow rate is decreased to 5 liters/hr and a well mixed solution flows out at an increased rate of 5 liters/hr.

How much contaminate is in the tank 2 hours after the change?

6. Take the same situation from #5 and 2 hours after the change the inflow concentration is changed to a concentration of 10 grams/liter while the flow rate is kept at 5 liters/hr and the well mixed solution flows out at an increased rate of 14 liters/hr. Set up, but do not solve, an IVP for this new situation.