1. (4 pts) Here's the IVP's we need for this problem. Note that I'm using a time frame of months here and so all per week quantities will need to be multiplied by 4 to get them into a per month quantity.

$$P' = rP \qquad P(0) = 250 \qquad P(4) = 1000$$

$$P' = rP - 6(4) - 12(4) = rP - 72 \qquad P(0) = 250$$

Solving the first and applying the initial condition gives the following solution which we can then apply the second condition,

$$P(t) = 250\mathbf{e}^{rt} \qquad 1000 = 250\mathbf{e}^{4r} \qquad r = \tfrac{1}{4}\ln(4)$$

The second IVP is now,

$$P' = \tfrac{1}{4}\ln(4)P - 72 \qquad P(0) = 250$$

I'll leave it to you to verify that the solution is,

$$P(t) = \tfrac{288}{\ln 4} + 42.2519\mathbf{e}^{\frac{1}{4}\ln(4)t}$$

From this we can see that the insects will survive because everything is positive and the exponential will go to infinity as $t \to \infty$.

3. (3 pts) Here's the IVP for this case.

$$v' = 9.8 - \tfrac{2}{12}v = 9.8 - \tfrac{1}{6}v \qquad v(0) = 0.2$$

I'll leave it to you to verify the solution to this.

$$v(t) = 58.8 - 58.6\mathbf{e}^{-\frac{1}{6}t}$$

The height function is,

$$s(t) = \int 58.8 - 58.6\mathbf{e}^{-\frac{1}{6}t}\,dt \qquad s(0) = 0 \qquad \Rightarrow \qquad s(t) = 58.8t + 351.6\mathbf{e}^{-\frac{1}{6}t} - 351.6$$

We know the velocity the object hits the ground we can determine when the object hits the ground.

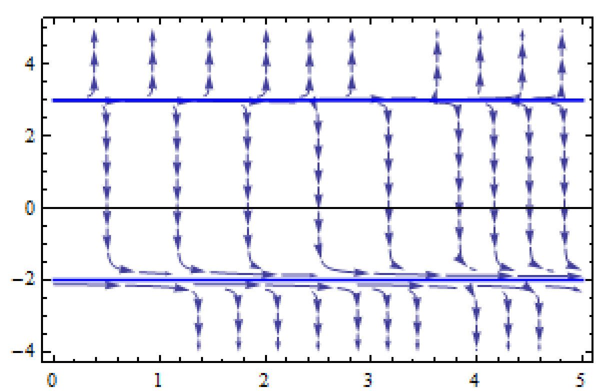
$$30 = 58.8 - 58.6\mathbf{e}^{-\frac{1}{6}t} \qquad \to \qquad \mathbf{e}^{-\frac{1}{6}t} = 0.4915 \qquad \Rightarrow \qquad t = 4.2622$$

The bridge is then $s(4.2622) = 71.8148\,m$ above the ground.

5. (3 pts) The equilibrium solutions are $y = 3$ and $y = -2$. A sketch of some solutions is to the right and from this we see the following classifications.

$$y = 3 \qquad \text{Unstable}$$

$$y = -2 \qquad \text{Semi-stable}$$



Not Graded

2. For this case the IVP is,

$$P' = rP - 6(4) - 12(4) - 8(4) = rP - 104 \qquad\qquad P(2) = 292.2519$$

I'll leave it to you to verify that the solution is then,

$$P(t) = \tfrac{416}{\ln 4} - 3.9143\mathbf{e}^{\frac{1}{4}\ln(4)t}$$

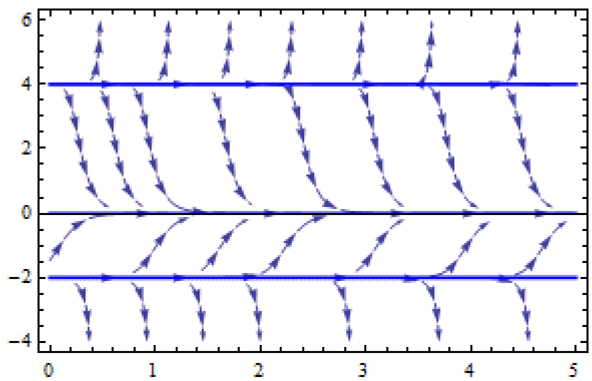
Okay, the coefficient of the exponential is negative and so we can see that the fish will die out. They die out at,

$$0 = \tfrac{416}{\ln 4} - 3.9143\mathbf{e}^{\frac{1}{4}\ln(4)t} \quad \to \quad \mathbf{e}^{\frac{1}{4}\ln(4)t} = 76.6626 \quad \Rightarrow \quad \boxed{t = 12.5209}$$

So, the well die out after 12.5209 months.

4. The equilibrium solutions are $y = 0$, $y = 4$ and $y = -2$. A sketch of some solutions is to the right and from this we see the following classifications.

$y = 4$	Unstable
$y = 0$	Asympt. Stable
$y = -2$	Unstable



6. We just need to run through the formulas using $f(t,y) = t\,y - \mathbf{e}^{t-y}$. Here's the results for $h = 0.3$.

t	0.3	0.6
f_n	-0.1353352832	0.3975666155
Approx.	1.9593994150	2.0786693997

Here's the results for $h = 0.15$.

t	0.15	0.3	0.45	0.6
f_n	-0.1353352832	0.1364932101	0.4174003100	0.7289207274
Approx.	1.9796997075	2.0001736890	2.0627837355	2.1721218446

For $h = 0.3$ we have $y(0.6) \approx 2.0786693997$ and for $h = 0.15$ we have $y(0.6) \approx 2.1721218446$.