4. (4 pts) $r^2+3r-18=(r-3)(r+6)=0 \quad \to \quad r_1=3,\ r_2=-6 \quad \to \quad \underline{y(t)=c_1\mathbf{e}^{3t}+c_2\mathbf{e}^{-6t}}$

$$\begin{aligned} c_1+c_2&=4+3\beta \\ 3c_1-6c_2&=1-\beta^2 \end{aligned} \quad \to \quad \begin{aligned} c_1&=-\tfrac{\beta^2-18\beta-25}{9} \\ c_2&=\tfrac{\beta^2+9\beta+11}{9} \end{aligned} \quad \to \quad y(t)=-\tfrac{\beta^2-18\beta-25}{9}\mathbf{e}^{3t}+\tfrac{\beta^2+9\beta+11}{9}\mathbf{e}^{-6t}$$

The second term will go to zero as $t\to\infty$ and the exponential in the first term will go to ∞ as $t\to\infty$. So, in order for the solution to remain finite the coefficient of the first term must be zero or,

$$\beta^2-18\beta-25=0 \quad \Rightarrow \quad \boxed{\beta=9\pm\sqrt{106}}$$

6. (3 pts) $r^2-8r+20=0 \quad \to \quad r_{1,2}=4\pm 2i \quad \to \quad \underline{y(t)=c_1\mathbf{e}^{4t}\cos(2t)+c_2\mathbf{e}^{4t}\sin(2t)}$

$$\begin{aligned} -c_1&=0 \\ -4c_1\mathbf{e}^{2\pi}-2c_2\mathbf{e}^{2\pi}&=7 \end{aligned} \quad \to \quad \begin{aligned} c_1&=0 \\ c_2&=-\tfrac{7}{2}\mathbf{e}^{-2\pi} \end{aligned} \quad \to \quad \boxed{y(t)=-\tfrac{7}{2}\mathbf{e}^{4t-2\pi}\sin(2t)}$$

7. (3 pts) $r^2+18r+81=(r+9)^2=0 \quad \to \quad r_{1,2}=-9 \quad \to \quad \underline{y(t)=c_1\mathbf{e}^{-9t}+c_2t\mathbf{e}^{-9t}}$

$$\begin{aligned} c_1&=-5 \\ -9c_1+c_2&=2 \end{aligned} \quad \to \quad \begin{aligned} c_1&=-5 \\ c_2&=-43 \end{aligned} \quad \to \quad \boxed{y(t)=-5\mathbf{e}^{-9t}-43t\mathbf{e}^{-9t}}$$

Not Graded

1. $6r^2-r-4=0 \quad \to \quad r_{1,2}=\frac{1\pm\sqrt{97}}{12} \quad \to \quad \boxed{y(t)=c_1\mathbf{e}^{\frac{1+\sqrt{97}}{12}t}+c_2\mathbf{e}^{\frac{1-\sqrt{97}}{12}t}}$

2. $5r^2+3r-2=(5r-2)(r+1)=0 \quad \to \quad r_1=\frac{2}{5},\ r_2=-1 \quad \to \quad \underline{y(t)=c_1\mathbf{e}^{\frac{2}{5}t}+c_2\mathbf{e}^{-t}}$

$$\begin{aligned} c_1+c_2&=-1 \\ \tfrac{2}{5}c_1-c_2&=0 \end{aligned} \quad \to \quad \begin{aligned} c_1&=-\tfrac{5}{7} \\ c_2&=-\tfrac{2}{7} \end{aligned} \quad \to \quad \boxed{y(t)=-\tfrac{5}{7}\mathbf{e}^{\frac{2}{5}t}-\tfrac{2}{7}\mathbf{e}^{-t}}$$

3. $r^2-4=(r-2)(r+2)=0 \quad \to \quad r_1=2,\ r_2=-2 \quad \to \quad \underline{y(t)=c_1\mathbf{e}^{2t}+c_2\mathbf{e}^{-2t}}$

$$\begin{aligned} c_1+c_2&=4 \\ 2c_1-2c_2&=-7 \end{aligned} \quad \to \quad \begin{aligned} c_1&=\tfrac{1}{4} \\ c_2&=\tfrac{15}{4} \end{aligned} \quad \to \quad \boxed{y(t)=\tfrac{1}{4}\mathbf{e}^{2t}+\tfrac{15}{4}\mathbf{e}^{-2t}}$$

5. $9r^2-6r+10=0 \quad \to \quad r_{1,2}=\frac{1}{3}\pm i \quad \to \quad \underline{y(t)=c_1\mathbf{e}^{\frac{1}{3}t}\cos(t)+c_2\mathbf{e}^{\frac{1}{3}t}\sin(t)}$

$$\begin{aligned} c_1&=-3 \\ \tfrac{1}{3}c_1+c_2&=-1 \end{aligned} \quad \to \quad \begin{aligned} c_1&=-3 \\ c_2&=0 \end{aligned} \quad \to \quad \boxed{y(t)=-3\mathbf{e}^{\frac{1}{3}t}\cos(t)}$$

8. $4r^2-12r+9=(2r-3)^2=0 \quad \to \quad r_{1,2}=\frac{3}{2} \quad \to \quad \underline{y(t)=c_1\mathbf{e}^{\frac{3}{2}t}+c_2t\mathbf{e}^{\frac{3}{2}t}}$

$$\begin{aligned} c_1\mathbf{e}^6 + 4c_2\mathbf{e}^6 &= 0 \\ \tfrac{3}{2}c_1\mathbf{e}^6 + 7c_2\mathbf{e}^6 &= -9 \end{aligned} \quad\rightarrow\quad \begin{aligned} c_1 &= 36\mathbf{e}^{-6} \\ c_2 &= -9\mathbf{e}^{-6} \end{aligned} \quad\rightarrow\quad \boxed{y(t) = 36\mathbf{e}^{\frac{3}{2}t-6} - 9t\mathbf{e}^{\frac{3}{2}t-6}}$$