

Undetermined Coefficients, Part II

For problems 1 & 2 use the method of undetermined coefficients to determine the general solution to the given differential equation.

1. $y'' + 9y' = 6 + 2\cos(3t) - 9\sin(3t)$

2. $y'' - 6y' + 9y = e^{-t} + 4e^{3t}$

3. Solve the following IVP using the method of undetermined coefficients.

$$y'' - 4y' - 12y = 6t - 8e^{-2t} \quad y(0) = 6, \quad y'(0) = -1$$

For problems 4 & 5 write down the guess that we'd need to use with the method of undetermined coefficients to find the particular solution. Do not attempt to find the actual particular solution.

4. $y'' + y' - 56y = 6e^{-8t} + 5t - e^{12t} - (2 - 8t)e^{-8t}$

5. $y'' + 10y' + 29y = 6t\cos(2t) + (8 + 3t)e^{-5t}\sin(2t) - 10\sin(2t)$

Variation of Parameters

6. Use the method of variation of parameters to find the solution to the following differential equation.

$$y'' + 2y' + 5y = 15e^{-t}$$

7. Use the method of variation of parameters to find the solution to the following IVP.

$$9y'' - y = 10 - 3t \quad y(0) = 0, \quad y'(0) = 8$$

Vibrations

For problems 8 – 11 any solutions containing both a sine and a cosine must be combined into a single cosine. Any decimal work should be to at least the 4th decimal place.

8. A 4 lb object will stretch a spring 8 inches by itself. The mass has no damping and is initially displaced 4 inches upwards from its equilibrium position with an initial velocity of 10 in/sec upwards. Determine the displacement at any time t .

9. A 2 kilogram object will stretch a spring 40 cm by itself. The mass has a damper hooked up that will exert a force of 15 N when the velocity is 75 cm/sec. The mass is initially displaced 50 inches downwards from its equilibrium position with an initial velocity of 15 in/sec downwards. Determine the displacement at any time t . What kind of damping does the system experience?

10. Take the system from #8 and hook up a forcing function of the form $g(t) = 6\cos(5t) - \sin(5t)$ and determine the displacement at any time t . Will this system experience resonance?

11. Take the system from #9 and hook up a forcing function of the form $g(t) = 8\sin(2t)$. Determine the displacement at any time t .