Math 3301 **Homework Set 7 – Solutions** **10 Points**

1. (2 pts) $F(s) = \frac{6(4!)}{s^{4+1}} + \frac{4}{s-15} - 3\frac{(1)(3)(5)(7)\sqrt{\pi}}{2^4 s^{4+\frac{1}{2}}} = \boxed{\frac{144}{s^5} + \frac{4}{s-15} - \frac{315\sqrt{\pi}}{16 s^{\frac{9}{2}}}}$

4. (2 pts) $H(s) = 4\frac{2(9)s}{(s^2+81)^2} + \frac{s\cos(1) + 7\sin(1)}{s^2+49} = \boxed{\frac{72s}{(s^2+81)^2} + \frac{s\cos(1)+7\sin(1)}{s^2+49}}$

5. (2 pts)

$$G(s) = \frac{4-s}{3\left(s^2 - s + \frac{5}{3}\right)} = \frac{1}{3}\frac{4-\left(s-\frac{1}{2}+\frac{1}{2}\right)}{\left(s-\frac{1}{2}\right)^2 + \frac{17}{12}} = \frac{1}{3}\frac{4-\frac{1}{2}-\left(s-\frac{1}{2}\right)}{\left(s-\frac{1}{2}\right)^2+\frac{17}{12}} = \frac{1}{3}\left[\frac{\frac{7}{2}\left(\frac{\sqrt{17}}{\sqrt{12}}\frac{\sqrt{12}}{\sqrt{17}}\right)}{\left(s-\frac{1}{2}\right)^2+\frac{17}{12}} - \frac{s-\frac{1}{2}}{\left(s-\frac{1}{2}\right)^2+\frac{17}{12}}\right]$$

$$\boxed{g(t) = \tfrac{1}{3}\left(\tfrac{7\sqrt{12}}{2\sqrt{17}}\mathbf{e}^{\frac{1}{2}t}\sin\left(\tfrac{\sqrt{17}}{\sqrt{12}}t\right) - \mathbf{e}^{\frac{1}{2}t}\cos\left(\tfrac{\sqrt{17}}{\sqrt{12}}t\right)\right)}$$

6. (2 pts)

$$F(s) = \frac{A}{s+1} + \frac{B}{s-7} + \frac{C}{4s-3} \quad \to \quad 3+s = A(s-7)(4s-3) + B(s+1)(4s-3) + C(s+1)(s-7)$$

$$\begin{array}{lll} s=-1: \quad 2 = A(-8)(-7) & & A = \frac{1}{28} \\ s=7 \ : \quad 10 = B(8)(25) & \Rightarrow & B = \frac{1}{20} \\ s=\frac{3}{4} \ : \quad \frac{15}{4} = C\left(\frac{7}{4}\right)\left(-\frac{25}{4}\right) & & C = -\frac{12}{35} \end{array}$$

$$F(s) = \frac{\frac{1}{28}}{s+1} + \frac{\frac{1}{20}}{s-7} - \frac{\frac{12}{35}}{4\left(s-\frac{3}{4}\right)} \quad \to \quad \boxed{f(t) = \tfrac{1}{28}\mathbf{e}^{-t} + \tfrac{1}{20}\mathbf{e}^{7t} - \tfrac{3}{35}\mathbf{e}^{\frac{3}{4}t}}$$

9. (2 pts) $F(s) = \frac{A}{s-3} + \frac{Bs+C}{s^2+9} + \frac{Ds+E}{(s^2+9)^2}$

$$\begin{aligned} 8 &= A(s^2+9)^2 + (Bs+C)(s-3)(s^2+9) + (Ds+E)(s-3) \\ &= (A+B)s^4 + (-3B+C)s^3 + (18A+9B-3C+D)s^2 + (-27B+9C-3D+E)s + 81A - 27C - 3E \end{aligned}$$

$$\begin{array}{lrcl} s^4: & A+B=0 & & A = \frac{2}{81} \\ s^3: & -3B+C=0 & & B = -\frac{2}{81} \\ s^2: & 18A+9B-3C+D=0 & \to & C = -\frac{2}{27} \\ s^1: & -27B+9C-3D+E=0 & & D = -\frac{4}{9} \\ s^0: & 81A-27C-3E=8 & & E = -\frac{4}{3} \end{array}$$

$$F(s)=\frac{1}{81}\left[\frac{2}{s-3}-\frac{2s+6}{s^2+9}-\frac{36s+108}{\left(s^2+9\right)^2}\right]=\frac{1}{16}\left[\frac{2}{s-3}-\frac{2s}{s^2+9}-\frac{2(3)}{s^2+9}-\frac{6\left[2(3)s\right]}{\left(s^2+9\right)^2}-\frac{2\left[2\left(3^3\right)\right]}{\left(s^2+9\right)^2}\right]$$

$$f(t)=\boxed{\tfrac{1}{81}\left[2\mathbf{e}^{3t}-2\cos(3t)-2\sin(3t)-6t\sin(3t)-2\left(\sin(3t)-3t\cos(3t)\right)\right]}$$

Not Graded

2. $\boxed{H(s)=\dfrac{6s}{s^2+\frac{1}{49}}+\dfrac{4s}{s^2-49}}$

3. $\boxed{G(s)=\dfrac{2}{s^2+4}-\dfrac{26}{(s+8)^2+4}}$

7.

$$G(s)=\frac{A}{s}+\frac{B}{s^2}+\frac{C}{s^3}+\frac{D}{s+2}\quad\rightarrow\quad 1-s^2=As^2(s+2)+Bs(s+2)+C(s+2)+Ds^3$$

$$=(A+D)s^3+(2A+B)s^2+(2B+C)s+2C$$

$$\begin{aligned}s^3&:\quad A+D=0 & & A=-\tfrac{3}{8}\\ s^2&:\quad 2A+B=-1 & & B=-\tfrac{1}{4}\\ s^1&:\quad 2B+C=0 & \rightarrow\quad & C=\tfrac{1}{2}\\ s^0&:\quad 2C=1 & & D=\tfrac{3}{8}\end{aligned}$$

$$G(s)=\frac{-\frac{3}{8}}{s}-\frac{\frac{1}{4}}{s^2}+\frac{\frac{1}{2}\frac{2!}{2!}}{s^3}+\frac{\frac{3}{8}}{s+2}\quad\rightarrow\quad\boxed{g(t)=-\tfrac{3}{8}-\tfrac{1}{4}t+\tfrac{1}{4}t^2+\tfrac{3}{8}\mathbf{e}^{-2t}}$$

8.

$$H(s)=\frac{As+B}{s^2+4}+\frac{Cs+D}{s^2+6s+4}\qquad 4s+7=(As+B)\left(s^2+6s+4\right)+(Cs+D)\left(s^2+4\right)$$

$$=(A+C)s^3+(6A+B+D)s^2+(4A+6B+4C)s+4B+4D$$

$$\begin{aligned}s^3&:\quad A+C=0 & & A=-\tfrac{7}{24}\\ s^2&:\quad 6A+B+D=0 & & B=\tfrac{2}{3}\\ s^1&:\quad 4A+6B+4C=4 & \Rightarrow\quad & C=\tfrac{7}{24}\\ s^0&:\quad 4B+4D=7 & & D=\tfrac{13}{12}\end{aligned}$$

$$H(s)=\frac{1}{24}\left[\frac{-7s+16}{s^2+4}+\frac{7s+26}{s^2+6s+4}\right]=\frac{1}{24}\left[\frac{-7s+16}{s^2+4}+\frac{7(s+3-3)+26}{(s+3)^2-5}\right]$$

$$=\frac{1}{24}\left[\frac{-7s}{s^2+4}+\frac{(8)(2)}{s^2+4}+\frac{7(s+3)}{(s+3)^2-5}+\frac{\sqrt{5}\left(\sqrt{5}\right)}{(s+3)^2-5}\right]$$

$$h(t)=\boxed{\tfrac{1}{24}\left[-7\cos(2t)+8\sin(2t)+7\mathbf{e}^{-3t}\cosh\left(\sqrt{5}\,t\right)+\sqrt{5}\mathbf{e}^{-3t}\sinh\left(\sqrt{5}\,t\right)\right]}$$