

Step Functions

For problems 1 – 3 find the Laplace transform of the given function.

1. $g(t) = u_2(t) \sin\left(\frac{t}{6} - \frac{1}{3}\right) + 16u_4(t) e^{24-6t}$

2. $f(t) = u_8(t) \sin(2t) - 3t^2 u_3(t)$

3. $h(t) = \begin{cases} -3 & t < 6 \\ 4t & 6 \leq t < 11 \\ 1+t & t \geq 11 \end{cases}$

For problems 4 – 6 find the inverse Laplace transform of each of the following

4. $F(s) = \frac{e^{-8s}}{2s^2 + 14} - \frac{2e^{-s}}{s} - \frac{10e^{-3s}}{s^4}$

5. $F(s) = \frac{4s + 11e^{-4s} - se^{-9s}}{s(s-4)^2}$

6. $H(s) = \frac{6 - 7e^{-12s} + s^2 e^{-6s}}{(s-1)(s^2 + 3)}$

IVP's with Laplace Transforms

Use Laplace transforms to solve the given IVP. In the partial fraction stage all quadratics that can be factored with integer coefficients must be factored.

7. $y'' - 8y' + 65y = 10$ $y(0) = 1, y'(0) = -3$

8. $y'' - 11y' - 12y = 18t$ $y(0) = 0, y'(0) = 0$

9. $y'' + 9y = 8\cos(3t)$ $y(0) = 4, y'(0) = -1$