1. (2 pts) $g(t)=u_2(t)\sin\left(\frac{1}{6}(t-2)\right)+16u_4(t)\mathbf{e}^{-6(t-4)} \quad \to \quad \boxed{G(s)=\dfrac{\frac{1}{6}\mathbf{e}^{-2t}}{s^2+\frac{1}{36}}+\dfrac{16\mathbf{e}^{-4s}}{s+6}}$

3. (2 pts) First we'll need to write the function in terms of Heaviside functions and get things properly shifted.

$$\begin{aligned} h(t) &= -3+(3+4t)u_6(t)+(1-3t)u_{11}(t) \\ &= -3+\left(3+4(t-6+6)\right)u_6(t)+\left(1-3(t-11+11)\right)u_{11}(t) \\ &= -3+\left(27+4(t-6)\right)u_6(t)+\left(-32-3(t-11)\right)u_{11}(t) \end{aligned}$$

The transform is then,

$$\boxed{H(s)=-\frac{3}{s}+\mathbf{e}^{-6s}\left(\frac{27}{s}+\frac{4}{s^2}\right)+\mathbf{e}^{-11s}\left(-\frac{32}{s}-\frac{3}{s^2}\right)}$$

5. (2 pts)

$$F(s)=\frac{4s+11\mathbf{e}^{-4s}-s\mathbf{e}^{-9s}}{s(s-4)^2}=\left(4-\mathbf{e}^{-9s}\right)\frac{1}{(s-4)^2}+11\mathbf{e}^{-4s}\frac{1}{s(s-4)^2}=\left(4-\mathbf{e}^{-9s}\right)G(s)+11\mathbf{e}^{-4s}H(s)$$

Note that we'll only need to do partial fractions on the second term as the first is straight out of the table. I'll leave it to you to verify the partial fraction work.

$$G(s)=\frac{1}{(s-4)^2} \qquad \to \qquad \underline{g(t)=t\mathbf{e}^{4t}}$$

$$H(s)=\frac{1}{16}\left[\frac{1}{s}-\frac{1}{s-2}+\frac{4}{(s-4)^2}\right] \qquad \to \qquad \underline{h(t)=\tfrac{1}{16}\left(1-\mathbf{e}^{4t}+4t\mathbf{e}^{4t}\right)}$$

The inverse Laplace Transform is then,

$$F(s)=4G(s)-\mathbf{e}^{-9s}G(s)+11\mathbf{e}^{-4s}H(s)\to\boxed{f(t)=4g(t)-u_9(t)g(t-9)+11u_4(t)h(t-4)}$$

7. (2 pts) Take the Laplace transform of everything.

$$\begin{aligned} s^2Y(s)-sy(0)-y'(0)-8\left(sY(s)-y(0)\right)+65Y(s) &= \frac{10}{s} \\ \left(s^2-8s+65\right)Y(s)-s+11 &= \frac{10}{s} \\ \left(s^2-8s+65\right)Y(s) &= \frac{10}{s}+s-11=\frac{s^2-11s+10}{s} \\ Y(s) &= \frac{s^2-11s+10}{s\left(s^2-8s+65\right)} \end{aligned}$$

I'll leave it to you to verify the partial fraction work.

$$Y(s)=\frac{1}{13}\left[\frac{2}{s}+\frac{11s-127}{s^2-8s+65}\right]=\frac{1}{13}\left[\frac{2}{s}+\frac{11(s-4+4)-127}{(s-4)^2+49}\right]=\frac{1}{13}\left[\frac{2}{s}+\frac{11(s-4)}{(s-4)^2+49}-\frac{83\frac{7}{7}}{(s-4)^2+49}\right]$$

$$\boxed{y(t)=\tfrac{1}{13}\left(2+11\mathbf{e}^{4t}\cos(7t)-\tfrac{83}{7}\mathbf{e}^{4t}\sin(7t)\right)}$$

9. (2 pts) Take the Laplace transform of everything.

$$s^2Y(s)-sy(0)-y'(0)+9Y(s)=\frac{8s}{s^2+9}$$

$$(s^2+9)Y(s)-4s+1=\frac{8s}{s^2+9}$$

$$Y(s)=\frac{8s}{(s^2+9)^2}+\frac{4s-1}{s^2+9}$$

This is one of the rare times where no partial fractions are needed. Everything is straight out of the table. The solution is,

$$Y(s)=\frac{4(2)\left(\frac{3}{3}\right)s}{(s^2+9)^2}+\frac{4s}{s^2+9}-\frac{\frac{3}{3}}{s^2+9}\quad\to\quad\boxed{y(t)=\tfrac{4}{3}t\sin(3t)+4\cos(3t)-\tfrac{1}{3}\sin(3t)}$$

Not Graded

2. In this case neither of the two functions are properly shifted and so we'll need to do some work first.

$$\begin{aligned}f(t)&=u_8(t)\sin\left(2(t-8+8)\right)-3(t-3+3)^2u_3(t)\\&=u_8(t)\sin\left(2(t-8)+16\right)-3\left((t-3)^2+6(t-3)+9\right)u_3(t)\end{aligned}$$

Note that with the second term I just used the fact that $(a+b)^2=a^2+2ab+b^2$ with $a=t-3$, $b=3$. So, in the first case we're shifting $\sin(2t+16)$ and in the second we're shifting t^2+6t+9. The transform is then,

$$\boxed{F(s)=\frac{\mathbf{e}^{-8s}\left(s\sin(16)+2\sin(16)\right)}{s^2+4}+3\mathbf{e}^{-3s}\left(\frac{2}{s^3}+\frac{6}{s^2}+\frac{9}{s}\right)}$$

4. $F(s)=\dfrac{1}{2}\mathbf{e}^{-8s}\dfrac{\frac{\sqrt{7}}{\sqrt{7}}}{s^2+7}-\dfrac{2\mathbf{e}^{-s}}{s}-10\mathbf{e}^{-3s}\dfrac{\frac{3!}{3!}}{s^4}$

$$\boxed{f(t)=\tfrac{1}{2\sqrt{7}}u_8(t)\sin\left[\sqrt{7}(t-8)\right]-2u_1(t)-\tfrac{5}{3}(t-3)^3u_3(t)}$$

6. $H(s)=\left(6-7\mathbf{e}^{-12s}\right)\dfrac{1}{(s-1)(s^2+3)}+\mathbf{e}^{-6s}\dfrac{s^2}{(s-1)(s^2+3)}=\left(6-7\mathbf{e}^{-12s}\right)F(s)+\mathbf{e}^{-6s}G(s)$

I'll leave it to you to verify the partial fraction work.

$$F(s)=\frac{1}{4}\left[\frac{1}{s-1}-\frac{s+1}{s^2+3}\right]=\frac{1}{4}\left[\frac{1}{s-1}-\frac{s}{s^2+3}-\frac{1\frac{\sqrt{3}}{\sqrt{3}}}{s^2+3}\right] \rightarrow \underline{f(t)=\tfrac{1}{4}\left(\mathbf{e}^t-\cos\left(\sqrt{3}\,t\right)-\tfrac{1}{\sqrt{3}}\sin\left(\sqrt{3}\,t\right)\right)}$$

$$H(s)=\frac{1}{4}\left[\frac{1}{s-1}+\frac{3s+3}{s^2+3}\right]=\frac{1}{4}\left[\frac{1}{s-1}-\frac{3s}{s^2+3}-\frac{\sqrt{3}\sqrt{3}}{s^2+3}\right] \rightarrow \underline{h(t)=\tfrac{1}{4}\left(\mathbf{e}^t+3\cos\left(\sqrt{3}t\right)+\sqrt{3}\sin\left(\sqrt{3}t\right)\right)}$$

The inverse Laplace Transform is then,

$$H(s)=6F(s)-7\mathbf{e}^{-12s}F(s)+\mathbf{e}^{-6s}G(s) \rightarrow \boxed{h(t)=6f(t)-7u_{12}(t)f(t-12)+u_6(t)g(t-6)}$$

8. Take the Laplace transform of everything.

$$s^2Y(s)-sy(0)-y'(0)-11\left(sY(s)-y(0)\right)-12Y(s)=\frac{18}{s^2}$$

$$\left(s^2-11s-12\right)Y(s)=\frac{18}{s^2}$$

$$Y(s)=\frac{18}{s^2(s+1)(s-12)}$$

I'll leave it to you to verify the partial fraction work.

$$Y(s)=\frac{1}{104}\left[\frac{143}{s}-\frac{156}{s^2}-\frac{144}{s+1}+\frac{1}{s-12}\right] \rightarrow \boxed{y(t)=\tfrac{1}{104}\left(143-156t-144\mathbf{e}^{-t}+\mathbf{e}^{12t}\right)}$$