Math 3301 **Homework Set 9** **10 Points**

IVP's with Step Functions

Use Laplace transforms to solve the given IVP. In the partial fraction stage all quadratics that can be factored with integer coefficients must be factored.

1. $2y'' + 7y' - 4y = 6u_7(t)\mathbf{e}^{7-t}$ $\qquad y(0) = -1,\ y'(0) = 0$

2. $y'' + 2y' + 3y = u_4(t)\mathbf{e}^{4t-16} - 7u_{10}(t)$ $\qquad y(0) = 0,\ y'(0) = 0$

3. $y'' + 8y' + 16y = (4t - 32)u_8(t) - 7t$ $\qquad y(0) = 0,\ y'(0) = 5$

Dirac-Delta Function

Use Laplace transforms to solve the given IVP. In the partial fraction stage all quadratics that can be factored with integer coefficients must be factored.

4. $y'' - 3y' - 18y = 9\delta(t-8)$ $\qquad y(0) = -4,\ y'(0) = -1$

5. $2y'' - 7y' = 2\delta(t-1) + 14u_2(t)\mathbf{e}^{3t-6}$ $\qquad y(0) = 0,\ y'(0) = 0$

Convolution Integrals

6. Find the Laplace Transform of $f(t) = \int_0^t \tau^2 \mathbf{e}^{4\tau - 4t}\, d\tau$.

7. Use a convolution integral (make sure you evaluate the integral!) to find the inverse transform of

$$H(s) = \frac{6}{(s+1)(s-4)}$$

8. Find the solution to the following IVP in terms of $g(t)$.

$$y'' - 2y' + y = g(t) \qquad y(0) = 9,\ y'(0) = 4$$