

#3. (2 pts) This is really a combination of #1 and #2 and so won't have quite as much detail.

$$\sin(6x) = \frac{10}{13} \Rightarrow 6x = \sin^{-1}\left(\frac{10}{13}\right) = 0.8776$$

A quick look at a unit circle shows that a second angle is $\pi - 0.8776 = 2.2640$. The solutions are then,

$$6x = 0.8776 + 2\pi n \quad \& \quad 6x = 2.2640 + 2\pi n \quad n = 0, \pm 1, \pm 2, \dots$$

$$x = 0.1463 + \frac{\pi n}{3} \quad \& \quad x = 0.3773 + \frac{\pi n}{3}$$

$$n = 0 : x = \underline{0.1463}, \underline{0.3773} \quad n = 1 : x = \underline{1.1935}, \underline{1.4245}$$

We have the single underlined solution above.

#6. (2 pts) $x(3e^{2x-4} - 7e^{x-2x^2}) = 0 \Rightarrow x = 0, \quad 3e^{2x-4} - 7e^{x-2x^2} = 0$

Make sure you don't cancel the x! If you do you'll lose the $x = 0$ solution. For the second term all we need to do is move one of the exponentials to the other side and then divide by one of them (which we can do because neither will be zero!), merge them up into a single exponential and solve. Doing this gives,

$$3e^{2x-4} = 7e^{x-2x^2} \rightarrow \frac{e^{2x-4}}{e^{x-2x^2}} = \frac{7}{3} \rightarrow e^{2x^2+x-4} = \frac{7}{3} \rightarrow 2x^2 + x - 4 = \ln \frac{7}{3}$$

$$2x^2 + x - 4 - \ln \frac{7}{3} = 0 \rightarrow x = \frac{-1 \pm \sqrt{1 - 4(2)(-4 - \ln \frac{7}{3})}}{2(2)} = \frac{-1 \pm \sqrt{33 + 8 \ln \frac{7}{3}}}{4} = -1.8268, 1.3268$$

The two solutions are then : $x = 0$, $x = -1.8268$ and $x = 1.3268$.

#7. (2 pts)

$$\ln(2-3x) + \ln(4-x) = -1$$

$$\ln((2-3x)(4-x)) = -1 \Rightarrow 3x^2 - 14x + 8 = e^{-1} \Rightarrow 3x^2 - 14x + 8 - e^{-1} = 0$$

Now, this is an unpleasant looking quadratic, but it's easy to solve using the quadratic formula!

$$x = \frac{14 \pm \sqrt{(-14)^2 - 4(3)(8 - e^{-1})}}{2(3)} = \frac{14 \pm \sqrt{100 + 12e^{-1}}}{6} = \frac{7 \pm \sqrt{25 + 3e^{-1}}}{3} = \boxed{0.6303}, \underline{4.0364}$$

We exclude the second one because it will give logs of negative numbers if we plugged into the original equation.

#8. (2 pts) The average velocity function is,

$$a.v. = \frac{s(t) - s(\frac{1}{2})}{t - \frac{1}{2}} = \frac{\frac{1}{\pi} \sin(\pi t) - 16t^2 - (-4 + \frac{1}{\pi})}{t - \frac{1}{2}} = \frac{\frac{1}{\pi} \sin(\pi t) - 16t^2 + 4 - \frac{1}{\pi}}{t - \frac{1}{2}}$$

Now, simply plug in the given values of t to build the following table.

t	a.v.	t	a.v.
0.49	-15.82429333	0.51	-16.17570667
0.499	-15.98242920	0.501	-16.0175708
0.4999	-15.99824292	0.5001	-16.00175708
0.49999	-15.99982429	0.50001	-16.00017571

So, it looks like the instantaneous velocity at $t = \frac{1}{2}$ is -16 and this means that at $t = \frac{1}{2}$ the object is moving to the **left**.

#12. (2 pts) Here's the table of values for the function.

x	$g(x)$	x	$g(x)$
-0.1	-3.816201155	0.1	-4.011903778
-0.01	-3.989127344	0.01	-4.009122969
-0.001	-3.998991252	0.001	-4.000991248
-0.0001	-3.999899913	0.0001	-4.000099912

From this table of values it looks like we can estimate,

$$\lim_{x \rightarrow 0} \frac{\sin(6x) + 10x}{x^2 - 4x} = \boxed{-4}$$

Not Graded

#1. First solving for cosine gives : $\cos\left(\frac{x}{4}\right) = -\frac{2}{5}$ and $\cos^{-1}\left(-\frac{2}{5}\right) = 1.9823$. A quick look at the unit circle shows that a second angle is $2\pi - 1.9823 = 4.3009$ (or you could use -1.9823 if you don't mind negative values). The solutions are then,

$$\begin{aligned} \frac{x}{4} &= 1.9823 + 2\pi n \quad \& \quad \frac{x}{4} = 4.3009 + 2\pi n \\ \boxed{x = 7.9292 + 8\pi n \quad \& \quad x = 17.2036 + 8\pi n} \quad & n = 0, \pm 1, \pm 2, \dots \end{aligned}$$

#2. Now to get the solutions to this equation in the given interval we just need to take the solution from #1 and plug in values of n until we get all solutions.

$$\begin{aligned} n = -1 : x &= -\cancel{147.2035}, \underline{-7.9291} & n = 0 : x &= \underline{7.9292}, \underline{17.2036} \\ n = 1 : x &= \underline{33.0619}, \cancel{42.3363} \end{aligned}$$

So, we get the 4 underlined solutions above.

#4. This is really a combination of #1 and #2 and so won't have quite as much details.

$$\sin(2x) = -\frac{1}{3} \Rightarrow 2x = \sin^{-1}\left(-\frac{1}{3}\right) = -0.3398$$

A positive angle corresponding to this is $2\pi - 0.3398 = 5.9434$. You can use either one, but I'll use the positive one as I usually do. A quick look at a unit circle shows that a second angle is $\pi + 0.3398 = 2.8018$. The solutions are then,

$$\begin{aligned} 2x &= 2.8018 + 2\pi n \quad \& \quad 2x = 5.9434 + 2\pi n \\ \underline{x = 1.4009 + \pi n \quad \& \quad x = 2.9717 + \pi n} \quad & n = 0, \pm 1, \pm 2, \dots \end{aligned}$$

$$\begin{aligned} n = -1 : x &= -\cancel{1.7407}, \underline{-0.1699} & n = 0 : x &= \underline{1.4009}, \underline{2.9717} \\ n = 1 : x &= \cancel{4.5425}, \cancel{6.1133} \end{aligned}$$

So, we get the 3 underlined solutions above.

#5.

$$e^{z^2-6} = \frac{12}{5} \quad \Rightarrow \quad \ln(e^{z^2-6}) = z^2 - 6 = \ln\left(\frac{12}{5}\right) \quad \Rightarrow \quad \boxed{z = \pm\sqrt{6 + \ln\left(\frac{12}{5}\right)} = \pm 2.6221}$$

#9. The formula for m_{PQ} is: $m_{PQ} = \frac{f(x) - f(-4)}{x + 4} = \frac{10 \ln(1-x) + 7e^{8+2x} - (7 + 10 \ln(5))}{x + 4}$

Now, simply plug in the given values of x to build the following table.

x	m_{PQ}	x	m_{PQ}
-3.9	13.47792234	-4.1	10.70858455
-3.99	12.13893535	-4.01	11.86292602
-3.999	12.01380931	-4.001	11.9862093
-3.9999	12.00138009	-4.0001	11.99862009

So it looks like the slope at $x = -4$ is 12. The tangent line is then,

$$y = 7 + 10 \ln(5) + 12(x + 4) = 12x + 71.0944$$

#10. $\lim_{x \rightarrow -12} f(x) = 3$ means that as x gets closer and closer to $x = -12$, from both sides, the value of the function, $f(x)$, is getting closer and closer to 3. Since the limit does not care about what the function is actually doing at $x = -12$ we can't say that this means that $f(-12) = 3$. Remember that all a limit tells us is what a function is doing *around* the point in question.

#11. Here's the table of values for the function.

x	$f(x)$	x	$f(x)$
4.9	1.677966102	5.1	1.655737705
4.99	1.667779633	5.01	1.665557404
4.999	1.666777796	5.001	1.666555574
4.9999	1.666677778	5.0001	1.666655556

From this table of values it looks like we can estimate,

$$\lim_{x \rightarrow 5} \frac{x^2 - 25}{x^2 - 4x - 5} = \boxed{1.666\bar{6} = \frac{5}{3}}$$