

#1. (2 pts) $\lim_{x \rightarrow \infty} \frac{(x-7)(x-2)}{5x^2 + x - 10} = \lim_{x \rightarrow \infty} \frac{x^2 - 9x + 14}{5x^2 + x - 10} = \lim_{x \rightarrow \infty} \frac{x^2 \left(1 - \frac{9}{x} + \frac{14}{x^2}\right)}{x^2 \left(5 + \frac{1}{x} - \frac{10}{x^2}\right)} = \lim_{x \rightarrow \infty} \frac{1 - \frac{9}{x} + \frac{14}{x^2}}{5 + \frac{1}{x} - \frac{10}{x^2}} = \boxed{\frac{1}{5}}$

#4. (2 pts) Here's the work for the first limit.

$$\begin{aligned} \lim_{y \rightarrow \infty} \frac{\sqrt{6+2y^2}}{9y-8} &= \lim_{y \rightarrow \infty} \frac{\sqrt{y^2 \left(\frac{6}{y^2} + 2\right)}}{y \left(9 - \frac{8}{y}\right)} = \lim_{y \rightarrow \infty} \frac{|y| \sqrt{\frac{6}{y^2} + 2}}{y \left(9 - \frac{8}{y}\right)} \quad \text{b/c } y \rightarrow \infty \text{ we can assume } y > 0 \text{ so } |y| = y \\ &= \lim_{y \rightarrow \infty} \frac{y \sqrt{\frac{6}{y^2} + 2}}{y \left(9 - \frac{8}{y}\right)} = \lim_{y \rightarrow \infty} \frac{\sqrt{\frac{6}{y^2} + 2}}{9 - \frac{8}{y}} = \boxed{\frac{\sqrt{2}}{9}} \end{aligned}$$

Now, for the second limit the initial work is identical so we'll pick it up there.

$$\begin{aligned} \lim_{y \rightarrow \infty} \frac{\sqrt{6+2y^2}}{9y-8} &= \lim_{y \rightarrow \infty} \frac{|y| \sqrt{\frac{6}{y^2} + 2}}{y \left(9 - \frac{8}{y}\right)} \quad \text{b/c } y \rightarrow -\infty \text{ we can assume } y < 0 \text{ so } |y| = -y \\ &= \lim_{y \rightarrow \infty} \frac{-y \sqrt{\frac{6}{y^2} + 2}}{y \left(9 - \frac{8}{y}\right)} = \lim_{y \rightarrow \infty} \frac{-\sqrt{\frac{6}{y^2} + 2}}{9 - \frac{8}{y}} = \boxed{-\frac{\sqrt{2}}{9}} \end{aligned}$$

#6. (2 pts) To do this problem let's note that $M = 0$ (because we're asking to determine if $h(x) = 0$) and then all we really need are the values of the function at the two endpoints of the interval and show that zero is between them.

$$h(-3) = 0.914463 \qquad h(0) = -1$$

So, we have, $h(0) < 0 < h(-3)$ so by the IVT we know that there is a number c such that,

$$-3 < c < 0 \quad \text{and} \quad h(c) = 0$$

Or, in other words, $h(x)$ has a root somewhere in the interval $[-3, 0]$.

#8. (2 pts)

$$\begin{aligned} g'(x) &= \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} = \lim_{h \rightarrow 0} \frac{4 - 15(x+h) - (4 - 15x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{4 - 15x - 15h - 4 + 15x}{h} = \lim_{h \rightarrow 0} \frac{-15h}{h} = \lim_{h \rightarrow 0} -15 = -15 \quad \Rightarrow \quad \boxed{g'(x) = -15} \end{aligned}$$

#9. (2 pts)

$$\begin{aligned}
 V'(t) &= \lim_{h \rightarrow 0} \frac{V(t+h) - V(t)}{h} = \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{2-(t+h)}{4+3(t+h)} - \frac{2-t}{4+3t} \right) \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{(4+3t)(2-(t+h)) - (2-t)(4+3(t+h))}{(4+3(t+h))(4+3t)} \right) \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{8+2t-4h-3t^2-3th - (8+2t+6h-3t^2-3th)}{(4+3(t+h))(4+3t)} \right) \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{-10h}{(4+3(t+h))(4+3t)} \right) = \lim_{h \rightarrow 0} \frac{-10}{(4+3(t+h))(4+3t)} \\
 &= \frac{-10}{(4+3t)^2} \Rightarrow \boxed{V'(t) = \frac{-10}{(4+3t)^2}}
 \end{aligned}$$

Not Graded

$$\text{\#2. } \lim_{t \rightarrow -\infty} \frac{7t^4 + 6t^3 + 8t}{5 + 8t^2 - 4t^3} = \lim_{t \rightarrow -\infty} \frac{t^3 \left(7t + 6 + \frac{8}{t^2} \right)}{t^3 \left(\frac{5}{t^3} + \frac{8}{t} - 4 \right)} = \lim_{t \rightarrow -\infty} \frac{7t + 6 + \frac{8}{t^2}}{\frac{5}{t^3} + \frac{8}{t} - 4} = \frac{-\infty}{-4} = \boxed{\infty}$$

$$\text{\#3. } \lim_{z \rightarrow \infty} \frac{z^5 - 7z^3 + 5}{2z^6 + 4z^2} = \lim_{z \rightarrow \infty} \frac{z^6 \left(\frac{1}{z} - \frac{7}{z^3} + \frac{5}{z^6} \right)}{z^6 \left(2 + \frac{4}{z^4} \right)} = \lim_{z \rightarrow \infty} \frac{\frac{1}{z} - \frac{7}{z^3} + \frac{5}{z^6}}{2 + \frac{4}{z^4}} = \frac{0}{2} = \boxed{0}$$

#5. This function will not be continuous where the denominator is zero, so all we need to do is determine where the denominator is zero and we'll know where the function isn't continuous.

$$1 + \csc(2x) = 0 \quad \Rightarrow \quad \frac{1}{\sin(2x)} = -1 \quad \Rightarrow \quad \sin(2x) = -1$$

As we can see above it is easier to do this in terms of sine instead of cosecants. The solution is then,

$$2x = \frac{3\pi}{2} + 2\pi n \quad \Rightarrow \quad x = \frac{3\pi}{4} + \pi n \quad n = 0, \pm 1, \pm 2, \dots$$

Also note that we have division by zero in the cosecant itself, as we can see above. We also need to avoid where $\sin(2x) = 0$. The solution to this is,

$$\begin{aligned}
 2x &= 0 + 2\pi n & \Rightarrow & & x &= \pi n \\
 2x &= \pi + 2\pi n & & & x &= \frac{1}{2}\pi + \pi n
 \end{aligned} \quad n = 0, \pm 1, \pm 2, \dots$$

So, all the function will not be continuous at,

$$x = \pi n, \quad x = \frac{1}{2}\pi + \pi n, \quad x = \frac{3\pi}{4} + \pi n \quad n = 0, \pm 1, \pm 2, \dots$$

#7. This problem seems quite complicated at first. However, it's not as bad as it may seem. We are really being asked to find a range of no more than $\frac{1}{2}$ of a month inside of $[0, 5]$ where $P(t) = 2.5$ (remember the population is in hundreds.....). To start let's just get the population at all the integers in the interval $[0, 5]$. This won't give our answer, but it will allow us to get the answer.

$$\begin{array}{lll} P(0) = 4 & P(1) = 4.5181 & P(2) = 3.2996 \\ P(3) = 2.0250 & P(4) = 2.8113 & P(5) = 4.3395 \end{array}$$

Okay, there are two spots where the population is below 250. That means that somewhere in $[2, 3]$ and $[3, 4]$ the population will fall below 250. These are intervals of length 1, and we want intervals of length $\frac{1}{2}$. We can easily get these however by computing the following populations.

$$P(2.5) = 2.4795 \quad P(3.5) = 2.1571$$

So, it looks like the population will hit 250 in the either of the intervals,

$$[2, 2.5] \quad \text{or} \quad [3.5, 4]$$

Either of these are acceptable answers as they are both intervals of width no more than $\frac{1}{2}$.

#10.

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{4(x+h)^2 - 2(x+h) + 1 - (4x^2 - 2x + 1)}{h} \\ &= \lim_{h \rightarrow 0} \frac{4x^2 + 8xh + 4h^2 - 2x - 2h + 1 - 4x^2 + 2x - 1}{h} \\ &= \lim_{h \rightarrow 0} \frac{8xh + 4h^2 - 2h}{h} = \lim_{h \rightarrow 0} (8x + 4h - 2) = 8x - 2 \quad \Rightarrow \quad \boxed{f'(x) = 8x - 2} \end{aligned}$$

#11.

$$\begin{aligned} W'(z) &= \lim_{h \rightarrow 0} \frac{W(z+h) - W(z)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{1-4(z+h)} - \sqrt{1-4z}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{1-4(z+h)} - \sqrt{1-4z}}{h} \cdot \frac{\sqrt{1-4(z+h)} + \sqrt{1-4z}}{\sqrt{1-4(z+h)} + \sqrt{1-4z}} \\ &= \lim_{h \rightarrow 0} \frac{1-4z-4h-(1-4z)}{h(\sqrt{1-4(z+h)} + \sqrt{1-4z})} = \lim_{h \rightarrow 0} \frac{-4h}{h(\sqrt{1-4(z+h)} + \sqrt{1-4z})} \\ &= \lim_{h \rightarrow 0} \frac{-4}{\sqrt{1-4(z+h)} + \sqrt{1-4z}} = \frac{-4}{\sqrt{2+3z} + \sqrt{2+3z}} \quad \Rightarrow \quad \boxed{W'(z) = \frac{-2}{\sqrt{2+3z}}} \end{aligned}$$

#12. From #11 we know that $f'(x) = 8x - 2$ and $f'(-2) = -18 < 0$. So, the function must be **decreasing** at $x = -2$ because $f'(-2) < 0$.

#13. From #9 we know that $g'(x) = -15$ and so the slope of the tangent line is, $m = g'(9) = -15$. The tangent line is then,

$$y = g(9) + m(x - 9) = -131 - 15(x - 9) = -15x + 4 \Rightarrow \boxed{y = -15x + 4}$$

Note that the fact that the tangent line is the same as the original line shouldn't be too surprising here. A tangent line to a line will be the original line itself since it must be parallel to the line and touch the line at the given point, but any line that is parallel to another and touches the line can only mean that the two lines are the same line.

#14. From #10 we know that $V'(t) = \frac{-10}{(4 + 3t)^2}$. The function will only stop changing if its derivative is

zero, but the only way that a rational expression can be zero is if its numerator is zero and in our case that is impossible and so this function is **always changing**.

#15. Be careful in reading this problem. The graph is for the derivative and we're not asking if the graph (i.e. the derivative) is increasing/decreasing/not changing at the points. We're begin asked to determine if the function is increasing/decreasing/not changing at the points and we don't need the function itself to do that. We know that a function will be decreasing if its derivative is negative (i.e. the graph of the derivative is below the x-axis), the function will be increasing if its derivative is positive (i.e. the graph of the derivative is above the x-axis) and the function will not be changing if the derivative is zero (i.e. the graph of the derivative touches the x-axis). So, given all that the answers are,

$x = -2$: the function is decreasing because the graph is below the x-axis at this point.

$x = 1$: the function is decreasing because the graph is below the x-axis at this point.

$x = 4$: the function is not changing because the graph touches the x-axis at this point.