Homework Set 4

**#1.** (2 pts) 
$$f(t) = \frac{1}{12}t^{-5} - 3t^{-\frac{5}{4}} + 4t^{6} - 9t - 12 \qquad \Rightarrow \qquad f'(t) = \frac{-5}{12}t^{-6} + \frac{15}{4}t^{-\frac{9}{4}} + 24t^{5} - 9t^{-\frac{9}{4}} + 24t^{5} - 9t^{-\frac{9$$

(a) 
$$s'(t) = 6t^2 - 54t + 84 = 6(t-2)(t-7)$$
  
(b)  $6(t-2)(t-7) = 0 \implies t = 2, 7$ 

(c) We'll need to do a quick number line to check the signs of the derivative. I'll leave it to you to actually draw a number line, but here are the values of the derivatives that I used for the number line.

$$s'(1) = 36$$
  $s'(4) = -36$   $s'(8) = 36$ 

Here is the right/left information for this problem.

Right : 
$$[0,7), (7,\infty)$$
 Left :  $(2,7)$ 

## <u>#7.</u> (2 pts)

$$R(z) = \frac{3z^{2} + 4}{1 - 8z^{\frac{1}{2}}}$$
$$R'(z) = \frac{6z(1 - 8z^{\frac{1}{2}}) - (3z^{2} + 4)(-4z^{-\frac{1}{2}})}{(1 - 8z^{\frac{1}{2}})^{2}} = \boxed{\frac{6z - 36z^{\frac{3}{2}} + 16z^{-\frac{1}{2}}}{(1 - 8z^{\frac{1}{2}})^{2}}}$$

**#9.** (2 pts) 
$$f'(x) = -\csc(x)\cot(x) - 12x^5 \cot(x) + 2x^6 \csc^2(x)$$

## <u>#13.</u> (2 pts)

 $P'(t) = 5 + 9\cos(t) \rightarrow 5 + 9\cos(t) = 0 \rightarrow \cos(t) = -\frac{5}{9} \rightarrow t = \cos^{-1}(-\frac{5}{9}) = 2.1598$ 

A quick look at a unit circle shows that the second angle is  $2\pi - 2.1598 = 4.1234$ . All solutions are,

$$t = 2.1598 + 2\pi n$$
 &  $t = 4.1234 + 2\pi n$   $n = 0, \pm 1, \pm 2, ...$ 

Now, the solutions in [0, 15] are,

$$n = 0$$
 :  $t = 2.1598$ ,  $4.1234$   
 $n = 1$  :  $t = 8.4430$ ,  $10.4066$   
 $n = 2$  :  $x = 14.7262$ ,  $16.6898$ 

So, in [0, 15] the function is not changing at the 5 underlined solutions above.

## Not Graded

**#2.** 
$$g(x) = \frac{4}{\sqrt{x}} - \frac{2x}{\sqrt{x}} = 4x^{-\frac{1}{2}} - 2x^{\frac{1}{2}} \implies g'(x) = -2x^{-\frac{3}{2}} - x^{-\frac{1}{2}} = -\frac{2}{x^{\frac{3}{2}}} - \frac{1}{x^{\frac{1}{2}}}$$
  
Now we can get the tangent line.

 $g(16) = -7 \qquad g'(16) = -\frac{9}{32} \qquad \qquad \boxed{y = -7 - \frac{9}{32}(x - 16) = -\frac{6}{32}x - \frac{5}{2}}$ 

**#3.** We'll first need the derivative, but to do that we'll need to multiply the function out.

$$f(x) = -3x^{3} + 4x^{2} + 6x - 8 \qquad f'(x) = -9x^{2} + 8x + 6$$

If the tangent line is going to by perpendicular to the given line then it must have slope,

$$m = -\frac{1}{\frac{1}{4}} = -4$$

So, to find the point(s) we're after all we need to do is solve,

$$-4 = -9x^{2} + 8x + 6 \quad \Rightarrow \quad 9x^{2} - 8x - 10 = 0 \quad \Rightarrow \quad x = \frac{4 \pm \sqrt{106}}{9} = -0.6995, \ 1.5884$$

There are then two points that will give a perpendicular tangent line.

**<u>#5.</u>** This is really #4 without me stepping you through it. First we'll need the derivative and then we'll need to know where the derivative is zero. After we have this we'll do a number line for [-2, 6].

$$f'(x) = 5x^4 - 8x^3 - 42x^2 = x^2(5x^2 - 8x - 42)$$
$$x^2(5x^2 - 8x - 42) = 0 \implies x = 0, \frac{4 \pm \sqrt{226}}{5} = 0, -2.2067, 3.8067$$

Make sure you can deal with the quadratic formula. You'll be seeing them on most of the next few homework sets and potentially on exams!

Now, for this problem we don't actually need -2.2067 since the interval starts at -2. So doing a number line that is restricted to [-2, 6]. I'll leave it to you to verify the number line. Here are the derivative values that I used.

$$f'(-1) = -29$$
  $f'(1) = -45$   $f'(4) = 96$ 

So, we have the following increasing/decreasing information.

Decreasing : [-2,0), (0,3.8067) Increasing : (3.8067,6]

We didn't need the intervals of decreasing but it was just as easy to give them. So in [-2, 6] we are increasing for a total of 6 - 3.8067 = 2.1933. over the total distance of 8. This means that we're increasing for,

$$\frac{2.1933}{8} \times 100 = 27.42\%$$

<u>#6.</u>

$$y' = (-12x^{-3} - 7)(1 + 8x^{-1}) + (6x^{-2} - 7x + 3)(-8x^{-2}) = \boxed{(-12x^{-3} - 7)(1 + 8x^{-1}) - 8x^{-2}(6x^{-2} - 7x + 3)(-8x^{-2})} = \boxed{(-12x^{-3} - 7)(1 + 8x^{-1}) - 8x^{-2}(6x^{-2} - 7x + 3)(-8x^{-2})} = \boxed{(-12x^{-3} - 7)(1 + 8x^{-1}) - 8x^{-2}(6x^{-2} - 7x + 3)(-8x^{-2})} = \boxed{(-12x^{-3} - 7)(1 + 8x^{-1}) - 8x^{-2}(6x^{-2} - 7x + 3)(-8x^{-2})} = \boxed{(-12x^{-3} - 7)(1 + 8x^{-1}) - 8x^{-2}(6x^{-2} - 7x + 3)(-8x^{-2})} = \boxed{(-12x^{-3} - 7)(1 + 8x^{-1}) - 8x^{-2}(6x^{-2} - 7x + 3)}$$

**<u>#8.</u>** What we need here is the increasing/decreasing information for the interval [0, 8].

$$V'(t) = \frac{(-1)(4t^2 - 2t + 1) - (3 - t)(8t - 2)}{(4t^2 - 2t + 1)^2} = \frac{4t^2 - 24t + 5}{(4t^2 - 2t + 1)^2}$$

Recall that to determine where a rational expression is zero all we need to do is determine where the numerator is zero.

$$4t^2 - 24t + 5 = 0 \qquad \Rightarrow \qquad t = \frac{6 \pm \sqrt{31}}{2} = 0.2161, 5.7839$$

I'll let you verify the number line for this problem. Here are the values of the derivative that I used.

$$V'(0) = 5$$
  $V'(1) = -\frac{5}{3}$   $V'(6) = \frac{5}{17689}$ 

From this we get we can see that the battery is in use for for 5.7839 - 0.2161 = 5.5678 days and charging for

8 – 5.5678 = 2.4322 days. Therefore, **yes** the battery is in use more than it's being charged in [0, 10].

<u>#10.</u>

$$R'(\theta) = \frac{\left(3\theta^2 + 6\sec^2\right)\left(1 - \sec\theta\right) - \left(\theta^3 + 6\tan\theta\right)\left(-\sec\theta\tan\theta\right)}{\left(1 - \sec\theta\right)^2}$$
$$= \frac{\left[\frac{\left(3\theta^2 + 6\sec^2\right)\left(1 - \sec\theta\right) + \left(\theta^3 + 6\tan\theta\right)\left(\sec\theta\tan\theta\right)}{\left(1 - \sec\theta\right)^2}\right]}{\left(1 - \sec\theta\right)^2}$$

<u>#11.</u>

$$h'(z) = \frac{-3\sec^2 z (1 - 2\sin z) + 2\cos z (4 - 3\tan z)}{(1 - 2\sin z)^2}$$
$$h'(2) = \frac{-3\sec^2 (2) (1 - 2\sin(2)) + 2\cos(2) (4 - 3\tan(2))}{(1 - 2\sin(2))^2} = 8.0521 > 0$$

So, at z = 2 the function is **increasing** because the h'(2) > 0.

## Homework Set 4

10 Points

**#12.** 
$$y' = -\sin(x)\cot(x) - \cos(x)\csc^2(x) = -\cos(x) - \cos(x)\csc^2(x)$$
  
 $y|_{\pi} = \cos(\frac{\pi}{2})\cot(\frac{\pi}{2}) = 0$   $y'_{\pi}| = -\cos(\frac{\pi}{2}) - \cos(\frac{\pi}{2})\csc^2(\frac{\pi}{2}) = 0$ 

Don't get excited about all the zeroes. It just means that the tangent line is : y = 0.