#2. (2 pts)

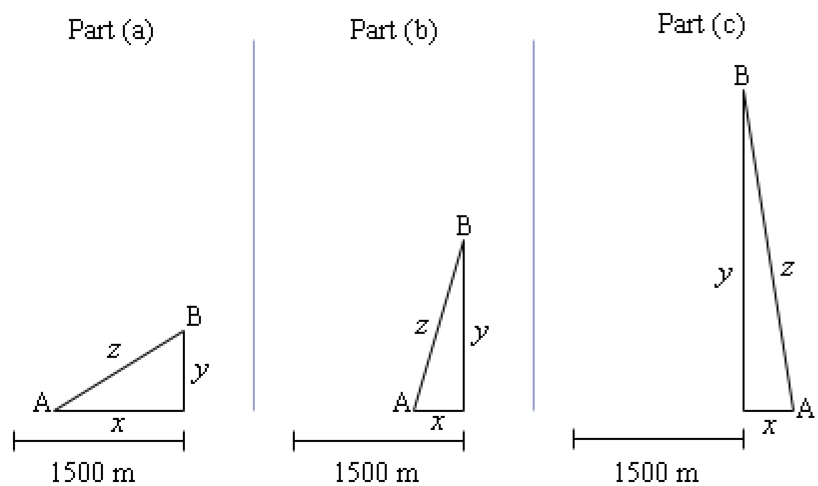
$$\left(18y^2y' - 2x\right)\mathbf{e}^{6y^3-x^2} = 8 - \sin\left(y\right)y'$$

$$18y^2\mathbf{e}^{6y^3-x^2}y' - 2x\mathbf{e}^{6y^3-x^2} = 8 - \sin\left(y\right)y'$$

$$\left(18y^2\mathbf{e}^{6y^3-x^2} + \sin\left(y\right)\right)y' = 8 + 2x\mathbf{e}^{6y^3-x^2}$$

$$\boxed{y' = \frac{8 + 2x\mathbf{e}^{6y^3-x^2}}{18y^2\mathbf{e}^{6y^3-x^2} + \sin\left(y\right)}}$$

#6. (4 pts) Here is the sketch for each part of this problem and notice that for **(c)** we've actually moved past the starting point of boat B.



In each case we're going to need to find z' and to do this we'll use the Pythagorean Theorem and so we may as well set that up now and then we'll actually work the problem.

$$x^2 + y^2 = z^2 \qquad \Rightarrow \qquad z' = \tfrac{1}{z}\left(x\,x' + y\,y'\right)$$

(a) Here's all the important quantities for this part.

$$x = 1500 - 30\left(2\right) = 1440 \qquad x' = -30 \qquad y = 60\left(2\right) = 120 \qquad y' = 60$$

$$z = \sqrt{1440^2 + 120^2} = \sqrt{1974400} = 1405.1334$$

The rate at which the distance between the two boats is changing is,

$$z' = \tfrac{1}{1405.1334}\left(\left(1440\right)\left(-30\right) + \left(120\right)\left(60\right)\right) = \underline{-25.6203 \text{ mph}}$$

So, in this case the distance is **decreasing**.

(b) Here's all the important quantities for this part.

$$x = 1500 - 30\left(10\right) = 1200 \qquad x' = -30 \qquad y = 60\left(10\right) = 600 \qquad y' = 60$$

$$z = \sqrt{1200^2 + 600^2} = \sqrt{1800000} = 1341.6408$$

The rate at which the distance between the two boats is changing is,

$$z' = \tfrac{1}{1341.6408}\left(\left(1200\right)\left(-30\right) + \left(600\right)\left(60\right)\right) = \underline{0 \text{ mph}}$$

So, in this case the distance is **not changing**.

(c) Here's all the important quantities for this part.

$$x = 30(75) - 1500 = 750 \qquad x' = 30 \qquad y = 60(75) = 4500 \qquad y' = 60$$

$$z = \sqrt{750^2 + 4500^2} = \sqrt{20812500} = 4562.0719$$

The rate at which the distance between the two boats is changing is,

$$z' = \tfrac{1}{4562.0719}\left((750)(30) + (4500)(60)\right) = \underline{64.1156 \text{ mph}}$$

So, in this case the distance is **increasing**.

#9. (2 pts) $y' = \dfrac{2\mathbf{e}^{2x}}{1+\mathbf{e}^{2x}} \qquad y'' = \dfrac{4\mathbf{e}^{2x}\left(1+\mathbf{e}^{2x}\right) - 2\mathbf{e}^{2x}\left(2\mathbf{e}^{2x}\right)}{\left(1+\mathbf{e}^{2x}\right)^2} = \boxed{\dfrac{4\mathbf{e}^{2x}}{\left(1+\mathbf{e}^{2x}\right)^2}}$

#10. (2 pts) Don't forget to simplify the derivative of the logarithm to make the rest of the work easier.

$$g'(x) = \frac{45x^2}{15x^3} - 4\mathbf{e}^{-4x} - 3\cos(3x) = 3x^{-1} - 4\mathbf{e}^{-4x} - 3\cos(3x)$$

$$g''(x) = -3x^{-2} + 16\mathbf{e}^{-4x} + 9\sin(3x) \qquad \boxed{g^{(3)}(x) = 6x^{-3} - 64\mathbf{e}^{-4x} + 27\cos(3x)}$$

Not Graded

#1.

$$2x\mathbf{e}^{x^2} - 8y^8 y' = 4x^3\sin(10y) + 10x^4\cos(10y)\, y'$$

$$\left(10x^4\cos(10y) + 8y^7\right)y' = 2x\mathbf{e}^{x^2} - 4x^3\sin(10y) \qquad \Rightarrow \qquad \boxed{y' = \frac{2x\mathbf{e}^{x^2} - 4x^3\sin(10y)}{10x^4\cos(10y) + 8y^7}}$$

#3. First we need y' then we can find the equation of the tangent line.

$$\cos\left(x^2y^4\right)\left(2xy^4 + 4x^2y^3y'\right) = 6y'x^3 + 18yx^2$$

$$2xy^4\cos\left(x^2y^4\right) + 4x^2y^3y'\cos\left(x^2y^4\right) = 6y'x^3 + 18yx^2$$

$$\left(4x^2y^3\cos\left(x^2y^4\right) - 6x^3\right)y' = 18yx^2 - 2xy^4\cos\left(x^2y^4\right) \qquad \boxed{y' = \frac{18yx^2 - 2xy^4\cos\left(x^2y^4\right)}{4x^2y^3\cos\left(x^2y^4\right) - 6x^3}}$$

Now the tangent line.

$$y'\big|_{x=-2,\ y=0} = \frac{0}{48} = 0 \qquad \Rightarrow \qquad y = 0 + 0(x+2) \qquad \Rightarrow \qquad \boxed{y = 0}$$

Don't get excited about this being the tangent line. Sometimes they end up looking like this!

#4. There are two formulas we'll need here, the volume and surface area of a sphere.

$$V = \tfrac{4}{3}\pi r^3 \qquad\qquad S = 4\pi r^2$$

We've been given $V' = -6$ (negative because the volume is decreasing) and want to find r' when

$$S = 48\pi \qquad \Rightarrow \qquad 48\pi = 4\pi r^2 \qquad \Rightarrow \qquad r = \sqrt{12} = 2\sqrt{3}$$

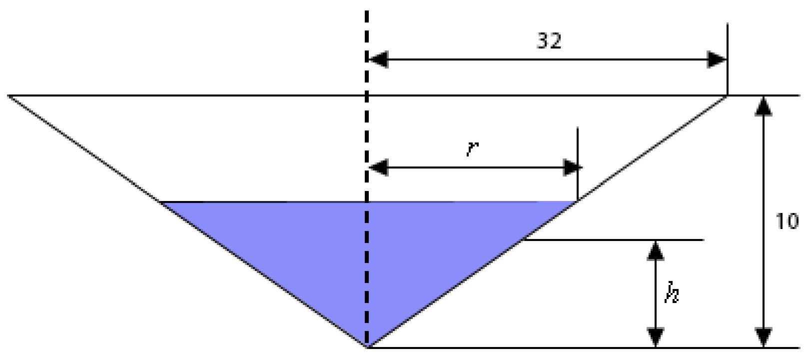
At this point all we really need to do is differentiate the volume formula and plug in the knows values.

$$V' = 4\pi r^2 r' \qquad \Rightarrow \qquad -6 = 4\pi\left(\sqrt{12}\right)^2 r' \qquad \Rightarrow \qquad \boxed{r' = -\frac{1}{8\pi}\ \text{cm/sec}}$$

#5. We'll need the volume of the tank since we know that $V' = 3.5$ (postive because the volume is increasing) and we want to determine h' when $h = 3$ so here's that equation.

$$V = \tfrac{1}{3}\pi r^2 h$$

Next, we'll need to relate *r* and *h* to make this easier to deal with. Here is a side view of the tank showing the two similar triangles we'll use to do this.



Using the two similar triangles we get the following relationship between *r* and *h*.

$$\frac{r}{h} = \frac{32}{10} \qquad \Rightarrow \qquad r = \frac{16}{5}h$$

The volume and its derivative is then,

$$V = \tfrac{1}{3}\pi\left(\tfrac{16}{5}h\right)^2 h = \tfrac{256}{75}\pi h^3 \qquad \Rightarrow \qquad V' = \tfrac{256}{75}\pi h^2 h'$$

All we need to do now is plug in the known quantities and solve.

$$3.5 = \tfrac{256}{75}\pi\left(3\right)^2 h' \qquad \Rightarrow \qquad \boxed{h' = 0.03627\ \text{ft/min}}$$

#7. $B(x) = B(x) = x^{\frac{3}{5}} - \tfrac{1}{2}x^{-1} + 9x \qquad B'(x) = \tfrac{3}{5}x^{-\frac{2}{5}} + \tfrac{1}{2}x^{-2} + 9 \qquad \boxed{B''(x) = -\tfrac{6}{25}x^{-\frac{7}{5}} - x^{-3}}$

#8.

$$f'(t) = -3\sin\left(4+3t\right) - 9\left(10t\right)\mathbf{e}^{5t^2} \qquad \boxed{f''(t) = -9\cos\left(4+3t\right) - 90\mathbf{e}^{5t^2} - 900t^2\mathbf{e}^{5t^2}}$$