Math 2413 **Homework Set 7** **10 Points**

#1. (2 pts)

$$f'(y)=4(12-2y)^3(-2)(y^2-3)^2+(12-2y)^4(2)(y^2-3)(2y)$$
$$=4(12-2y)^3(y^2-3)\left[-2(y^2-3)+y(12-2y)\right]=\underline{4(12-2y)^3(y^2-3)\left[-4y^2+12y+6\right]}$$

Now, to get the critical points we'll need to solve the following three equations,

$$\begin{aligned}12-2y&=0\\ y^2-3&=0\\ 4y^2-12y-6&=0\end{aligned}\qquad\Rightarrow\qquad\boxed{\begin{aligned}&y=6\\ &y=\pm\sqrt{3}=\pm1.7321\\ &y=\tfrac{12\pm\sqrt{240}}{8}=-0.4365,\ 3.4365\end{aligned}}$$

So, we have a total of 5 critical points for this function.

#2. (2 pts)

$$G'(z)=6z\left(7-z^2\right)^{\frac{1}{3}}+\left(3z^2+1\right)\left(\tfrac{1}{3}\right)\left(7-z^2\right)^{-\frac{2}{3}}(-2z)=6z\left(7-z^2\right)^{\frac{1}{3}}-\frac{2z\left(3z^2+1\right)}{3\left(7-z^2\right)^{\frac{2}{3}}}$$
$$=\frac{18z\left(7-z^2\right)-2z\left(3z^2+1\right)}{3\left(7-z^2\right)^{\frac{2}{3}}}=\frac{4z\left(31-6z^2\right)}{3\left(7-z^2\right)^{\frac{2}{3}}}$$

From the denominator we see that $\boxed{z=\pm\sqrt{7}}$ must be a critical point since the derivative won't exist at those points (but the function does!). Also, the derivative will be zero at,

$$\boxed{z=0,\quad z=\pm\sqrt{\tfrac{31}{6}}=\pm2.2730}$$

This function has five critical points.

#8. (2 pts) We found the critical points to this function in #3 and those that are in [-4, -1] are : -2.2749. So, all we need to do is evaluate the function at this point and the end points.

$$Q(-4)=-376\qquad Q(-2.2749)=158.9892\qquad Q(-1)=65$$

The absolute maximum is 158.9892 at $t=-2.2749$ and the absolute minimum is -376 at $t=-4$.

#10. (2 pts) We'll first need the critical points of this function that lie in [0,15].

$$P'(t)=8-50\frac{2t}{t^2+6}=\frac{8t^2-100t+48}{t^2+6}=\frac{4(2t-1)(t-12)}{t^2+6}$$

So, there are two critical points for this function, $t=\frac{1}{2}$ and $t=12$ and both lie in the interval so all we need to do now is some quick function evaluations.

$$P(0)=160.4120\qquad P\left(\tfrac{1}{2}\right)=162.3709\qquad P(12)=95.4682\qquad P(15)=97.8791$$

The maximum population is then 16,237.09 (fractions don't make much sense, but I'll keep them) at $t=\frac{1}{2}$ and the minimum population is 9,546.82 at $t=12$.

#13. **(2 pts)** $f'(t) = 15t^4 - 20t^3 - 360t^2 = 5x^2(3x^2 - 4x - 72)$ So, it looks like the critical points are,

$$x = 0, \tfrac{2 \pm \sqrt{55}}{3} = -4.2775,\ 5.6108$$

I will leave it to you to actually draw a number line for this problem. Here are the derivative values that I used for mine.

$$f'(-5) = 2875 \qquad f'(-1) = -325 \qquad f'(1) = -365 \qquad f'(6) = 2160$$

From this we get the following increasing/decreasing information and classification of critical points.

$$\text{Increasing : } (-\infty, -4.2775),\ (5.6108, \infty) \qquad \text{Decreasing : } (-4.2775, 0),\ (0, 5.6108)$$

$$x = -4.2775 : \text{Rel. Max.} \qquad x = 0 : \text{Neither} \qquad x = 5.6108 : \text{Rel. Min.}$$

Not Graded

#3. $Q'(t) = -12t^3 + 36t^2 + 144t = -12t(t^2 - 3t - 12) \qquad \Rightarrow \qquad \boxed{t = 0, \tfrac{3 \pm \sqrt{57}}{2} = 0, -2.2749, 5.2749}$

#4. $A'(t) = 5 - 21\cos(3t) \quad \Rightarrow \quad \cos(3t) = \tfrac{5}{21} \quad \Rightarrow \quad 3t = \cos^{-1}\left(\tfrac{5}{21}\right) = 1.3304$

$$3t = 1.3304 + 2\pi n \qquad \& \qquad 3t = 4.9528 + 2\pi n \qquad n = 0, \pm 1, \pm 2, \ldots$$

$$\boxed{t = 0.4435 + \tfrac{2\pi n}{3} \qquad \& \qquad t = 1.6509 + \tfrac{2\pi n}{3} \qquad n = 0, \pm 1, \pm 2, \ldots}$$

#5. $h'(x) = 2x\mathbf{e}^{x^2-5} - 24x\mathbf{e}^{-4x^2} = 2x\left(\mathbf{e}^{x^2-5} - 12\mathbf{e}^{-4x^2}\right)$

Now, clearly $\boxed{x = 0}$ is a critical point. Any others will come from solving,

$$\mathbf{e}^{x^2-5} - 12\mathbf{e}^{-4x^2} = 0 \quad \Rightarrow \quad \mathbf{e}^{x^2-5} = 12\mathbf{e}^{-4x^2} \quad \Rightarrow \quad \mathbf{e}^{5x^2-5} = 12$$

$$\Rightarrow \quad 5x^2 - 5 = \ln(12) \quad \Rightarrow \quad \boxed{x = \pm\sqrt{\tfrac{1}{5}(12)} = \pm 1.2235}$$

So, there are three critical points for this function.

#6. Rel. Max : *b*, *d*, *f* Rel. Min. : *c*, *e* Abs. Max. : *d* Abs. Min. : *g*

#7. Not much to this problem. We found all the critical points in #1 and those that are in [1, 4] are : $y = \sqrt{3}$ and $y = 3.4365$. So, all we need to do is evaluate the function at these points and the end points.

$$f(\sqrt{3}) = 0 \qquad f(3.4365) = 53{,}623.9707$$

$$f(1) = 40{,}000 \qquad f(4) = 43{,}264$$

The absolute maximum is then 53,623.9707 at $y = 3.4365$ and the absolute minimum is 0 at $y = -2$ and $y = \sqrt{3}$.

#9. All we are really asking here is what is the absolute maximum of this function in [0, 3] and we know how to do that. In #4 we found the critical points of this function to be,

$$t = 0.4435 + \tfrac{2\pi n}{3} \qquad \& \qquad t = 1.6509 + \tfrac{2\pi n}{3} \qquad n = 0, \pm 1, \pm 2, \ldots$$

and the ones that are in [0, 3] (the interval we're interested in here) are,

$$0.4435 \qquad 1.6509 \qquad 2.5379$$

The function evaluations for this problem are,

$$A(0) = 6 \qquad A(0.4435) = 1.4186 \qquad A(1.6509) = 21.0533$$
$$A(2.5379) = 11.8906 \qquad A(3) = 18.1152$$

So, the absolute maximum is 21.0533, and this corresponds, to \$21,053.3 and so **yes**, the manager does get a bonus.

#11. In this problem we're asking for the absolute minimum of the function in [0, 5]. From #3 we know that we'll have two critical points in this region ($t = 0, 4$) and we need to evaluate the function at these points and the endpoints. Here are all the relevant evaluations for this problem.

$$Q(0) = 8 \qquad Q(4) = 648 \qquad Q(5) = 433$$

So, the absolute maximum of 648 and so it does rise above 500 grams and so the process will stop during the first 5 hours.

#12. In #5 we found the critical points of this function to be : $t = -1.2235,\ 0,\ 1.2235$. I will leave it to you to actually draw a number line for this problem. Here are the derivative values that I used.

$$h'(-2) = -1.4715 \qquad h'(-1) = 0.4029 \qquad h'(1) = -0.4029 \qquad h'(2) = 1.4715$$

From this we get the following increasing/decreasing information and classification of critical points.

$$\text{Increasing : } (-1.2235, 0),\ (1.2235, \infty) \qquad \text{Decreasing : } (-\infty, -1.2235),\ (0, 1.2235)$$
$$x = -1.2235 : \text{Rel. Min.} \qquad x = 0 : \text{Rel. Max.} \qquad x = 1.2235 : \text{Rel. Min.}$$