**The Shape of a Graph, Part II**

**1.** Determine the intervals where $g(x)=\frac{1}{5}x^6-\frac{9}{10}x^5-5x^4+9x-6$ is concave up and concave down. What are the inflection points for this function?

**2.** Use $f(t)=3t^5-5t^4-120t^3$ for this problem.

**(a)** What are the intervals of increase/decrease for this function?
**(b)** What are the relative extrema of this function?
**(c)** Find the intervals of concave up/concave down for this function.
**(d)** Find all the inflection points for this function.
**(e)** Use the information above to sketch the graph of this function.

**3.** Suppose that you know that the critical points of $f(x)$ are $x=-4$, $x=1$, and $x=8$ and that the second derivative is $f''(x)=4x^3-18x^2-46x+60$. If possible, classify the critical points. If it is not possible clearly explain why it is not possible.

**The Mean Value Theorem**

**4.** Verify that $g(x)=8x-\ln\left(x^2+1\right)-3$ satisfies the conditions of the Mean Value Theorem on [0, 1] and find all values of *c* that satisfy the conclusion of the Mean Value Theorem on [0, 1].

**5.** Suppose that we know that $f(x)$ is a continuous and differentiable function and that $f(-34)=f(-1)=9$. Show that $f(x)$ must have a critical point in the interval [-34, -1].

**Optimization**

In order to receive any credit for problems 6 – 8 you MUST use Calculus techniques to find the answer. Any decimal work should include at least 4 decimal places.

**6.** We want to construct a box whose base length is five times its base width and will have no lid. We have \$1200 to spend and the material for the sides cost \$10/ft$^2$ and the material for the bottom costs \$7/ft$^2$. Determine the dimensions of the box that will give the largest volume.

**7.** Find the point(s) on $x=3-y^2$ that are closest to the point (-1, 0).

**8.** We have a piece of wire that is 75 cm long and we're going to cut it into two pieces. One piece will be bent into a square and the other will be bent into a circle. Determine where the wire should be cut so that the enclosed areas will a maximum. Note that it is possible to have the whole piece of wire go to the square or to the circle (*i.e.* not cut it at all).

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**L'Hospitals Rule**

Evaluate each of the following limits.

**9.** $\lim_{t\to-\infty}\dfrac{5t^2+9t}{1+8t-2t^2}$

**10.** $\lim_{y\to3}\dfrac{3\mathbf{e}^{6-2y}+y^2-12}{y^2-6y+9}$

**11.** $\lim_{x\to\infty}\dfrac{2x-\mathbf{e}^{7x}}{3x+\mathbf{e}^{10x}}$

**12.** $\lim_{z\to\infty} z\ln\left(1+\dfrac{1}{2z}\right)$

**Differentials**

Compute the differential for each of the following functions.

**13.** $u=\sin\left(x^2-4x\right)$

**14.** $R=t^2\ln\left(6t\right)$