

**#1. (2 pts)**  $g'(x) = \frac{6}{5}x^5 - \frac{9}{2}x^4 - 20x^3 + 9$   $g''(x) = 6x^4 - 18x^3 - 60x^2 = 6x^2(x+2)(x-5)$

So, it looks like the possible inflection points are :  $x = -2, 0, 5$ . I'll leave it to you to sketch the number line for this problem. Here are the values of the second derivative I used.

$$g''(-3) = 432 \quad g''(-1) = -36 \quad g''(1) = -72 \quad g''(6) = 1728$$

From this we can get the following concavity information.

$$\text{Concave Down : } (-2, 0), (0, 5) \quad \text{Concave Up : } (-\infty, -2), (5, \infty)$$

We can also see that we have two inflection points :  $x = -2$  and  $x = 5$ .

**#4. (2 pts)** This function is both continuous everywhere as well as differentiable everywhere and so the conditions of the MVT are satisfied on  $[0, 1]$ . All we need to do at this point is plug into the conclusion of the MVT.

$$g'(c) = \frac{g(1) - g(0)}{1 - (0)}$$

$$8 - \frac{2c}{c^2 + 1} = \frac{4.3069 - (-3)}{1} = 7.3069$$

$$\frac{2c}{c^2 + 1} = 0.6931$$

$$2c = 0.6931(c^2 + 1)$$

$$0.6931c^2 - 2c + 0.6931 = 0 \quad \Rightarrow \quad c = \frac{2 \pm \sqrt{4 - 4(0.6931)(0.6931)}}{2(0.6931)} = \boxed{0.4027, \cancel{2.4828}}$$

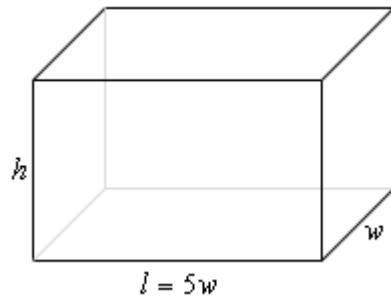
Note that the second does NOT satisfy the MVT as it does not fall in the interval  $[0, 1]$ .

**#6. (2 pts)** So, it looks like we'll be optimizing the volume and the cost will be the constraint. Here're the various equations we'll need.

$$V = lwh = 5w^2h$$

$$1200 = 7(wl) + 10[2(hw) + 2(lh)] = 35w^2 + 120wh$$

Solve the constraint for  $h$  and plug into the volume so we can differentiate and get the critical points.



$$h = \frac{1200 - 35w^2}{120w} = \frac{240 - 7w^2}{24w}$$

$$V(w) = 5w^2 \left( \frac{240 - 7w^2}{24w} \right) = \frac{5}{24} (240w - 7w^3) \quad V'(w) = \frac{5}{24} (240 - 21w^2) \quad V''(w) = -\frac{35}{4} w$$

So, it looks like the two critical points are :  $w = \pm\sqrt{\frac{240}{21}} = \pm 3.3806$  and since we're dealing with length here we only need to use the positive one. Plugging this into the second derivative shows us that, in fact, this gives a relative maximum and so is the correct value. The dimensions are then,

$$w = 3.3806 \quad l = 16.9030 \quad h = 1.9720$$

**#11. (2 pts)**  $\lim_{x \rightarrow \infty} \frac{2x - e^{7x}}{3x + e^{10x}} = \lim_{x \rightarrow \infty} \frac{2 - 7e^{7x}}{3 + 10e^{10x}} = \lim_{x \rightarrow \infty} \frac{-49e^{7x}}{100e^{10x}} = \lim_{x \rightarrow \infty} \left( -\frac{49}{100} e^{-3x} \right) = \boxed{0}$

**#12. (2 pts)**  $\lim_{z \rightarrow \infty} z \ln \left( 1 + \frac{1}{2z} \right) = \lim_{z \rightarrow \infty} \frac{\ln \left( 1 + \frac{1}{2z} \right)}{\frac{1}{z}} = \lim_{z \rightarrow \infty} \frac{-\frac{1}{2z^2}}{-\frac{1}{z^2}} = \lim_{z \rightarrow \infty} \frac{\frac{1}{2}}{1 + \frac{1}{2z}} = \boxed{\frac{1}{2}}$

**Not Graded**

**#2.** From #13 in the previous homework set we can get the answers to **(a)** and **(b)**.

Increasing :  $(-\infty, -4.2775), (5.6108, \infty)$       Decreasing :  $(-4.2775, 0), (0, 5.6108)$

$x = -4.2775$  : Rel. Max.       $x = 0$  : Neither       $x = 5.6108$  : Rel. Min.

Here's the remainder of the work for this problem.

$$f'(t) = 15t^4 - 20t^3 - 360t^2 \quad f''(t) = 60t^3 - 60t^2 - 720t = 60t(t+3)(t-4)$$

After setting the second derivative equal to zero and solving we get three possible inflection points,

$$t = -3, 0, 4$$

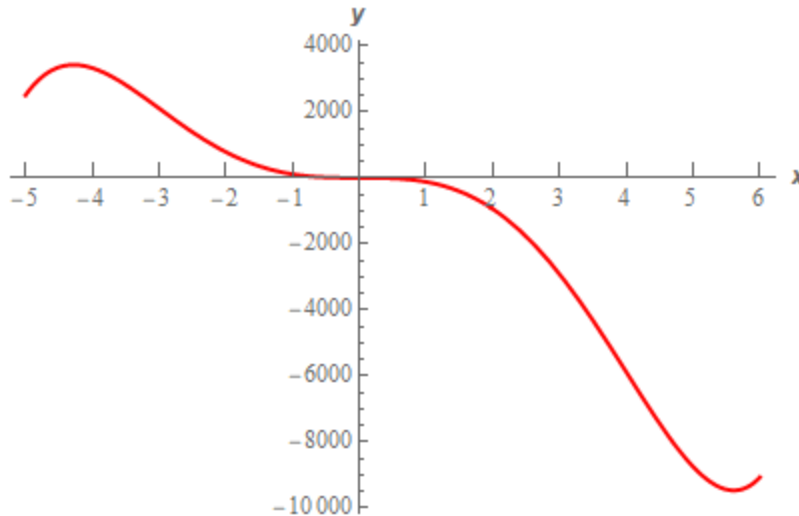
I'll leave it to you to sketch the number line for this problem. Here are the values of the second derivative I used.

$$g''(-4) = -1920 \quad g'(-1) = 600 \quad g''(1) = -720 \quad g''(5) = 2400$$

From this we can get the following concavity information.

Concave Down :  $(-\infty, -3), (0, 4)$       Concave Up :  $(-3, 0), (4, \infty)$

We can also see that we have three inflection points :  $t = -3$ ,  $t = 0$  and  $t = 4$ . Here's a sketch of the function on the interval  $[-5, 6]$ .



**#3.** This is simply an application of the second derivative test.

$$f''(-4) = -300 < 0 \quad \text{Relative Maximum}$$

$$f''(1) = 0 \quad \text{Don't know, 2}^{\text{nd}} \text{ derivative test says nothing here.}$$

$$f''(8) = 588 > 0 \quad \text{Relative Minimum}$$

Note that for  $x = 1$  the second derivative test tells us nothing about this critical point, it could be a relative minimum, relative, maximum or neither.

**#5.** Because we know that  $f(x)$  is a continuous and differentiable function we can apply the MVT as follows.

$$f'(c) = \frac{f(-1) - f(-34)}{-1 - (-34)} = \frac{9 - (9)}{33} = 0 \quad \Rightarrow \quad f'(c) = 0$$

So, by the MVT there is a  $c$  in  $[-34, -1]$  so that  $f'(c) = 0$  and so  $c$  is also a critical point of  $f(x)$ !

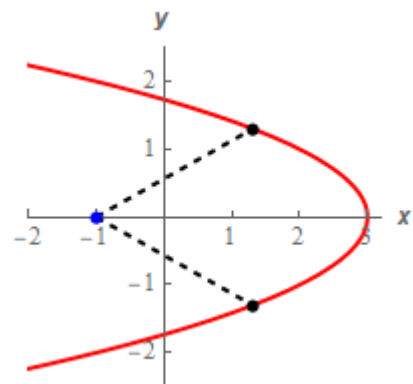
**#7.** In this case we want to minimize the square of the distance between  $(4, 0)$  and  $(x, y)$ , a point on the graph and the equation of the graph is the constraint.

$$d^2 = (x+1)^2 + y^2 \quad x = 3 - y^2$$

Now, solve the graph equation for  $y^2$  and plug this into the square of the distance to get the function we'll differentiate.

$$f(x) = (x+1)^2 + 3 - x = x^2 + x + 4 \quad f'(x) = 2x + 1$$

$$f''(x) = 2$$

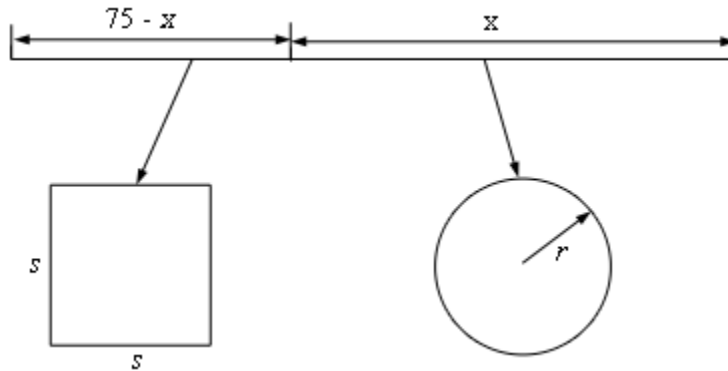


The only critical point we have is  $x = -\frac{1}{2}$  and it is a relative minimum by the second derivative test.

Now find  $y$  and we'll be done.

$$y^2 = 3 - \left(-\frac{1}{2}\right) = \frac{7}{2} \quad \rightarrow \quad y = \pm\sqrt{\frac{7}{2}} \quad \Rightarrow \quad \left(-\frac{1}{2}, \sqrt{\frac{7}{2}}\right), \left(-\frac{1}{2}, -\sqrt{\frac{7}{2}}\right)$$

**#8.** Here's a quick sketch of the situation.



Okay, here is what we know about these two objects and their perimeters compared to how much wire we used to make each object.

$$\text{Perimeter of Square : } 4s = 75 - x \quad \Rightarrow \quad s = \frac{1}{4}(75 - x)$$

$$\text{Circumference of circle : } 2\pi r = x \quad \Rightarrow \quad r = \frac{x}{2\pi}$$

$$\text{Total Length : } 75 = 4s + 2\pi r$$

There are two ways to proceed with solving this problem. One is to do the traditional optimization with constraint. Here are the equations that we'll need to deal with to do that.

$$\text{Maximize : } A = s^2 + \pi r^2$$

$$\text{Constraint : } 75 = 4s + 2\pi r$$

In this case we would solve the constraint for  $s$  or  $r$ , plug into the equation and maximize. Get  $s$  or  $r$  then determine what  $x$  needs to be.

The other way is to use the fact that the constraint is built into the perimeter and circumference formulas above and solve them in terms of  $x$  as I've done and plug these into the area and solve directly for  $x$ . In this case the area becomes

$$\begin{aligned} A(x) &= s^2 + \pi r^2 \\ &= \frac{1}{16}(75 - x)^2 + \pi \left(\frac{x}{2\pi}\right)^2 = \frac{1}{16}(75 - x)^2 + \frac{x^2}{4\pi} \end{aligned}$$

Now, if we just differentiate a couple of times we get,

$$A'(x) = -\frac{1}{8}(75 - x) + \frac{x}{2\pi} = \left(\frac{1}{2\pi} + \frac{1}{8}\right)x - \frac{75}{8}$$

$$A''(x) = \frac{1}{8} + \frac{1}{2\pi}$$

The only critical point in this case is

$$x = \frac{\frac{75}{8}}{\frac{1}{8} + \frac{1}{2\pi}} = 32.9926$$

Now, we've got a small problem here. The second derivative is always positive so that means that this critical point is in fact a relative minimum. In other words, if we use 32.9926 cm's for the circle and 42.0074 cm's for the square we will get the minimum enclosed area. This was not what the problem asked for however. So, it looks like what we need is to consider using all the wire for the square or all the wire for the circle. So, here are the areas if we use all the wire for the square and all the wire for the circle.

$$\text{Square : } A = \left(\frac{75}{4}\right)^2 = 351.5626 \quad \text{Circle : } A = \pi \left(\frac{75}{2\pi}\right)^2 = 447.6233$$

Don't forget that you'll need to determine the length of the side (wire length divided by 4 to get equal sides) and radius ( $70 = \text{Circumference}$ ) before doing these computations!

So, it looks like we'll need to use all the wire for the circle if we want to maximize the area.

$$\text{\#9. } \lim_{t \rightarrow -\infty} \frac{5t^2 + 9t}{1 + 8t - 2t^2} = \lim_{t \rightarrow -\infty} \frac{10t + 9}{8 - 4t} = \lim_{t \rightarrow -\infty} \frac{10}{-4} = \boxed{-\frac{5}{2}}$$

$$\text{\#10. } \lim_{y \rightarrow 3} \frac{3e^{6-2y} + y^2 - 12}{y^2 - 6y + 9} = \lim_{y \rightarrow 3} \frac{-6e^{6-2y} + 2y}{2y - 6} = \lim_{y \rightarrow 3} \frac{12e^{6-2y} + 2}{2} = \frac{14}{2} = \boxed{7}$$

$$\text{\#13. } \boxed{du = (2x - 4)\cos(x^2 - 4x)dx}$$

$$\text{\#14. } \boxed{dR = [2t \ln(6t) + t] dt}$$